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OPTIMAL FEEDBACK CONTROL OF TURBO-GENERATOR

by

© SAMUEL YOO

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled OPTIMAL FEEDBACK CONTROL OF TURBO-GENERATOR submitted by SAMUEL YOO in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

In this thesis the optimal torque and voltage control of a turbo-generator connected to an infinite bus is considered.

The generator is controlled through a linear feedback of the state variables. The feedback parameters are obtained by solving a two point nonlinear boundary value problem. The values to be obtained for these parameters depend on the strength and duration of the disturbance since the model is nonlinear contrary to the usual feedback control of a linear model.

The methods of solution are to cast the system into the Pontryagin's minimum principle and the gradient descent method. The model used includes the transfer functions of the governor, the turbine and the exciter.

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Table of Contents

		Page
Chapter I	Introduction	1
1.1	Preliminary Remarks	1
1.2	Scope of the Thesis	2
Chapter II	Background of Optimal Control	5
2.1	Introduction	5
2.2	Calculus of Variations	6
2.3	Pontryagin's Minimum Principle	11
2.4	Direct Computational Method	15
Chapter III	Modelling of the Synchronous Machine	20
3.1	Park's Transformation	20
3.2	Flux Linkage Equation	25
3.3	Torque Equation	28
Chapter IV	Mathematical Model of Turbo-Generator	36
4.1	Machine Equations in State Space	36
4.2	Modelling of the Turbine, the Governer and the Exciter	38
Chapter V	Mathematical Solution of the Model	50
5.1	System Equations in State Space	50
5.2	Mathematical Solution of the Model	54

		Page
Chapter VI	Computations of the System Equations	65
6.1	Computations for Turbo-Generator	65
6.2	Discussion of Results	70
Chapter V11	Conclusions	89
Bibliography		90
Appendices		93

List of Tables

Table	Description	Page
1-11	Tables Without Optimal Controls to K = 0.5	105
12-22	Tables With Optimal Controls to K = 0.5	116

List of Figures

Figure	Title	Page
1.1	An Organization of Power System Problems	3
2.1.1	Diagram of Control System	5
2.3.1	Free End Point Case	14
3.1	Pictorial Representation of a Synchronous Machine	20
3.3	Single-Line Diagram	30
4.2.1	Block Diagram of Steam Turbine	39
4.2.2.1	Functional Elements of a Speed Governor	41
4.2.2.2	Governor Droop	42
4.2.2.3	Block Diagram and Transfer Function for a Steam Turbine Governor	43
4.2.2.4	Block Diagram of a Steam Turbine Governor	44
4.2.3.1	Excitation Arrangements for a Synchronous Generator	45
4.2.3.2	The Effect of a Dead Band in Automatic Voltage Regulator	46
4.2.3.3	Block Diagram of a Continuously Acting Automatic Voltage Regulator	47
4.2.3.4	Block Diagram of Exciter	48
5.1	Schematic Diagram of the System Invest-igated	50
5.2	State-Space Diagram of the Model	56
6.1	Rotor-Angle vs. Time (at K = 0.4)	75
6.2	Rotor-Angle vs. Time (at K = 1.0)	76

Figure	Title	Page
6.3	Rotor-Angle vs. Time (at K = 2.1)	77
6.4	Phase Plane Plot at K = 2.1	78
6.5	Rotor-Angle vs. Time (at K = 0.1)	79
6.6	Rotor-Angle vs. Time (at K = 0.4)	80
6.7	Phase Plane Plot at K = 0.4	81
6.8	Optimum Control to Turbo-Generator Model (K = 0.4)	82
6.9	Optimum Control to Turbo-Generator Model (K = 0.4)	82
6.10	Variation of Optimum Field Voltage (K = 0.4)	83
6.11	Variation of Optimum Mechanical Torque (K = 0.4)	83
6.12	Variation of Optimum Angular Velocity (K = 0.4)	84
6.13	Variation of Optimum Excitation Voltage (K = 0.4)	85
6.14	Rotor-Angle vs. Time (at K = 0.5)	86
6.15	Optimal Parameters vs. Disturbance	87
6.16	Optimal Parameters vs. Disturbance	88

CHAPTER I

INTRODUCTION

1.1 Preliminary Remarks

Modern trends in power systems towards the use of generating units of larger capacity and lower specific inertia, and longer transmission lines at higher voltages tend to decrease the transient stability and steady state stability limits of the system. The problem of improving the steady state stability has been studied and the various methods proposed have been shown to provide an adequate margin of stability in the leading power factor region of operation even under low loads.

As the steady state stability limits are constantly improved, the operation of the power system tends to become increasingly constrained by its transient stability limits. Methods which have been proposed for increasing the transient stability margin act generally by controlling the output of the generator during transient conditions.

Regulation of the power input to the prime-mover will undoubtedly produce a direct effect on the rotor swings of the generator under transient conditions and can be achieved in the case of a thermal set by control of the steam valve. However, due to the presence of entrained steam storage effects in the various parts of the turbine, in the reheater and in the connecting pipes, the maximum benefit from this type of control are not generally achieved. Moreover, the centrifugal type of governor and control equipment are designed mainly to prevent overspeeding of the turbine under conditions of major load rejection or line switching.

From these observations, to obtain the benefits of turbine regulation during the transient period, it is necessary to consider its effect simultaneously with that of the automatic voltage regulator.

The method of approach is through the use of dynamic optimization techniques to determine an optimal feedback control policy subject to a prescribed performance criterion. For this purpose either the calculus of variations or the Pontryagin's minimum principle could be used. The criterion for optimum transient performance is taken as an integral quadratic index which includes soft constraints on the control variables.

1.2 Scope of the Thesis

In this thesis a general nonlinear model of a turbo-alternator connected to an infinite bus is considered. Control of the turbo-alternator is effected through control of field voltage and turbine torque. In addition, it is proposed to include the transfer functions of the governer, the turbine and the voltage regulator. Inclusion of these transfer functions provides a more realistic model of the machine and its control system, however, the dimensions of the system are increased.

The machine is disturbed from the steady state by a torque pulse applied directly to the shaft and while the alternator is controlled through a linear feedback of the state variables. The controls are modelled as linear combinations of the state variables, but the machine equations are not linearized. Therefore, the model parameters which are assumed to be time-independent will generally depend on the strength and duration of the pulse contrary to the linearized machine models where the equations themselves are linearized.

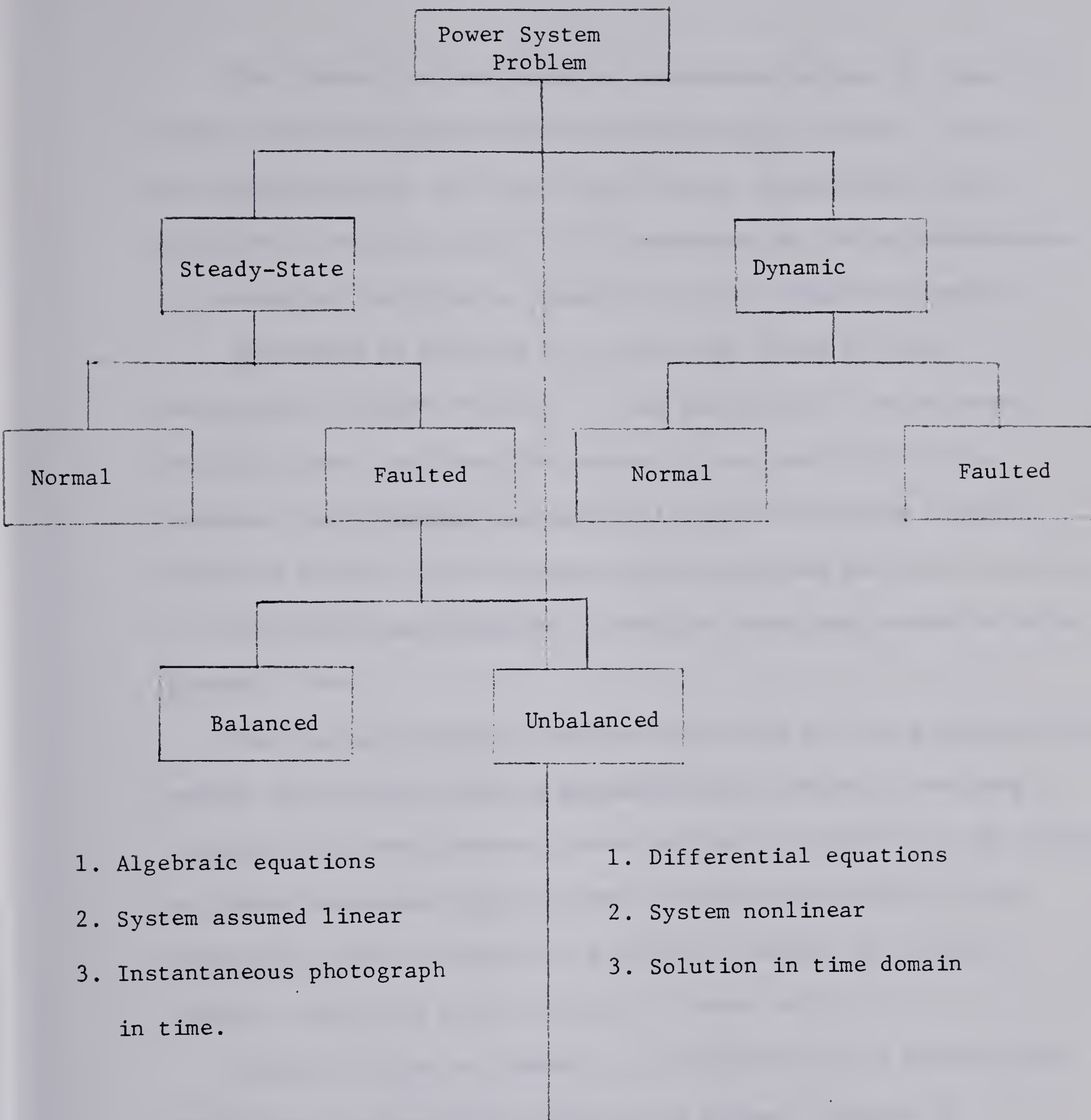

Fig. 1.1 An Organization of Power System Problems

This linear feedback model is considered because it leads to a simple implementation compared to the open loop system. Due to the restrictions on the form of the control as mentioned above, additional constraints have to be considered in the implementations of the optimal solution as compared with the unrealistic model.

The method of solution is to cast the system into the Pontryagin's minimum principle. This ensures that the necessary and sufficient conditions for optimality are met [1] which is important for obtaining the numerical solution for such a highly nonlinear system. The nonlinear system equations and the constrains on the feedback are augmented to the cost functional which is in the quadratic form.

Then using Pontryagin's minimum principle and the gradient descent method, the optimal feedback parameters are obtained by solving a two point nonlinear boundary value problem (T.P.N.B.V.P.). The values obtained for these parameters depend on the strength and duration of the disturbance since the model is nonlinear contrary to the usual feedback control of a linear model as shown in Fig. 1.1

Using this kind of feedback, ill-conditioning is handled more easily when the numerical value of the optimal solution is obtained.

CHAPTER II

BACKGROUND OF OPTIMAL CONTROL

2.1 Introduction

In the classical control system design, acceptable performance is generally defined in terms of time domain criteria such as rise time, settling time, and peak overshoot; and frequency domain criteria such as gain margin, phase margin and bandwidth. The tools used are frequency response, root locus phase plane and describing function, although these methods are widely used, they can be applied only to time-invariant systems, with unconstrained variables.

Modern systems are multiple input-multiple output; constraints on state and control variables have to be imposed. The objective of optimal control theory can be stated as follows: Determine the control signals which will enable a process or a plant to satisfy certain specified physical constraints and at the same time optimize some index or measure of performance.

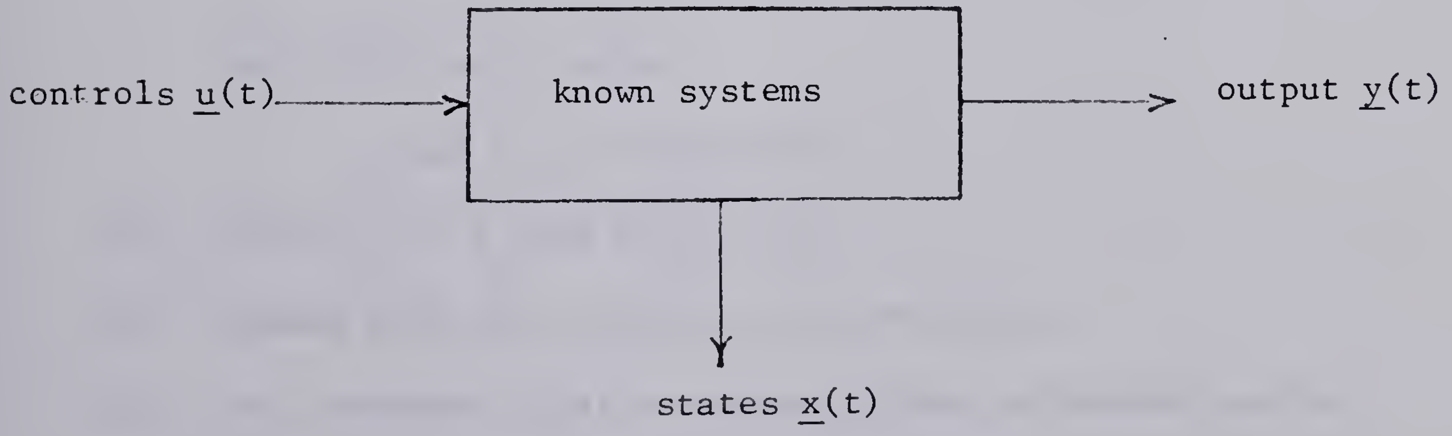

Figure 2.1.1 Diagram of Control System

The optimization problem can be expressed as:

Given a dynamical system as shown in Fig. 2.1.1,

State equation: $\dot{\underline{x}}(t) = \underline{f}[\underline{x}(t), \underline{u}(t), t]$

Output equation: $\underline{y}(t) = g[\underline{x}(t), \underline{u}(t), t]$

with (a) the initial state expressed as $\underline{x}(t_o)$ at $t=t_o$

(b) the final state expressed as $S(\underline{x}(t_f))$ where S is called the target S

(c) constraints on control variables expressed as $\underline{u}(t) \varepsilon U$

(d) a performance criterion or cost functional usually written as

$$J=J[\underline{x}(t_o), t_o, \underline{u}]$$

$$= \int_{t_o}^{t_f} L[\underline{x}(t), \underline{u}(t), t]dt$$

2.2 Calculus of Variations

2.2.1 Basic Notions of Calculus of Variations

PROBLEM

Determine the extremal $x^*(t)$ for the functional

$$J = \int_{t_o}^{t_f} L(x(t), \dot{x}(t), t)dt \qquad (2.2.1.1)$$

where $x(t)$ is a vector

t_o and t_f are specified.

(1) Let $x(t_o) = x_o$ and $x(t_f) = x_f$.

(2) Assume that $x(t)$ and $\dot{x}(t)$ are continuous.

(3) The integrand L has continuous first and second partial derivatives with respect to its arguments.

Consider an arbitrary function $\eta(t)$. Assume that η and $\dot{\eta}(t)$ are continuous in the closed interval $[t_o, t_f]$.

Let

$$x(t) = x^*(t) + \varepsilon\,\eta(t) \tag{2.2.1.2}$$

where $\varepsilon \geq 0$ is a real parameter and $x^*(t)$ is the solution. Substitute (2.2.1.2) into (2.2.1.1)

$$\begin{aligned} J(x) &= J[x^*(t) + \varepsilon\,\eta(t)] \\ &= J(\varepsilon) \\ &= \int_{t_o}^{t_f} L[t,\ x^*(t) + \varepsilon\,\eta(t),\ \dot{x}^*(t) + \varepsilon\,\dot{\eta}(t)]dt \end{aligned}$$

Since $x^*(t)$ extremizes J, we can say that J is a min. or max. when $x(t) = x^*(t)$ or when $\varepsilon = 0$. Considering J as a function of ε we set

$$\frac{dJ}{d\varepsilon}\Big|_{\varepsilon=o} = 0$$

to obtain the necessary conditions for $x^*(t)$ to be an extremal. That is

$$\frac{dJ}{d\varepsilon} = \int_{t_o}^{t_f} [L_t \frac{dt}{d\varepsilon} + L_{x^*+\varepsilon\eta} \cdot \eta + L_{\dot{x}^*+\varepsilon\dot{\eta}}\ \dot{\eta}]dt = 0$$

Since $\frac{dt}{d\varepsilon} = 0$, t does not depend on ε. The above can be simplified to yield at $\varepsilon = 0$,

$$\frac{dJ}{d\varepsilon} = \int_{t_o}^{t_f} L_{x^*} \cdot \eta\ dt + \int_{t_o}^{t_f} L_{\dot{x}^*} \cdot \dot{\eta}\ dt = 0 \tag{2.2.1.3}$$

If the second integral on the right hand side is integrated by parts

and substituted into (2.2.1.3), we obtain

$$\frac{dJ}{d\varepsilon} = \int_{t_o}^{t_f} (L_{x*} - \frac{d}{dt} L_{\dot{x}*})\ \eta(t)dt + L_{\dot{x}*}\ \eta(t) \Big|_{t_o}^{t_f} = 0 \qquad (2.2.1.4)$$

Since $\eta(t)$ is an arbitrary function, equation (2.2.1.4) will be satisfied only if the following necessary conditions are satisfied:

$$L_{x*} - \frac{d}{dt}\ L_{\dot{x}*} = 0$$

$$L_{\dot{x}*}\ \eta(t)\ \Big|_{t_o}^{t_f} = 0$$

These necessary conditions are shown in Fig. 2.2.1.

Since $x(t) = x*(t) + \varepsilon\eta(t)$ for all t in $[t_o, t_f]$ we have

$x(t_o) = x*(t_o) + \varepsilon\ \eta\ (t_o)$ and

$x(t_f) = x*(t_f) + \varepsilon\ \eta\ (t_f)$.

2.2.2 Variational Notation

We replace $x(t)$ by $x(t) + \varepsilon\eta(t)$ and represent $\varepsilon\eta(t)$ by $\delta x(t)$.

a. First Variation of a Functional $L(x,\dot{x},t)$

By definition

$$\delta L = L(x + \varepsilon\eta,\ \dot{x} + \varepsilon\dot{\eta},\ \ t) - L(x,\ \dot{x},\ t)$$

Expanding $L(x + \varepsilon\eta,\ \dot{x} + \varepsilon\dot{\eta},\ t)$ in a McLaurin's series in ε and retaining only the linear terms,

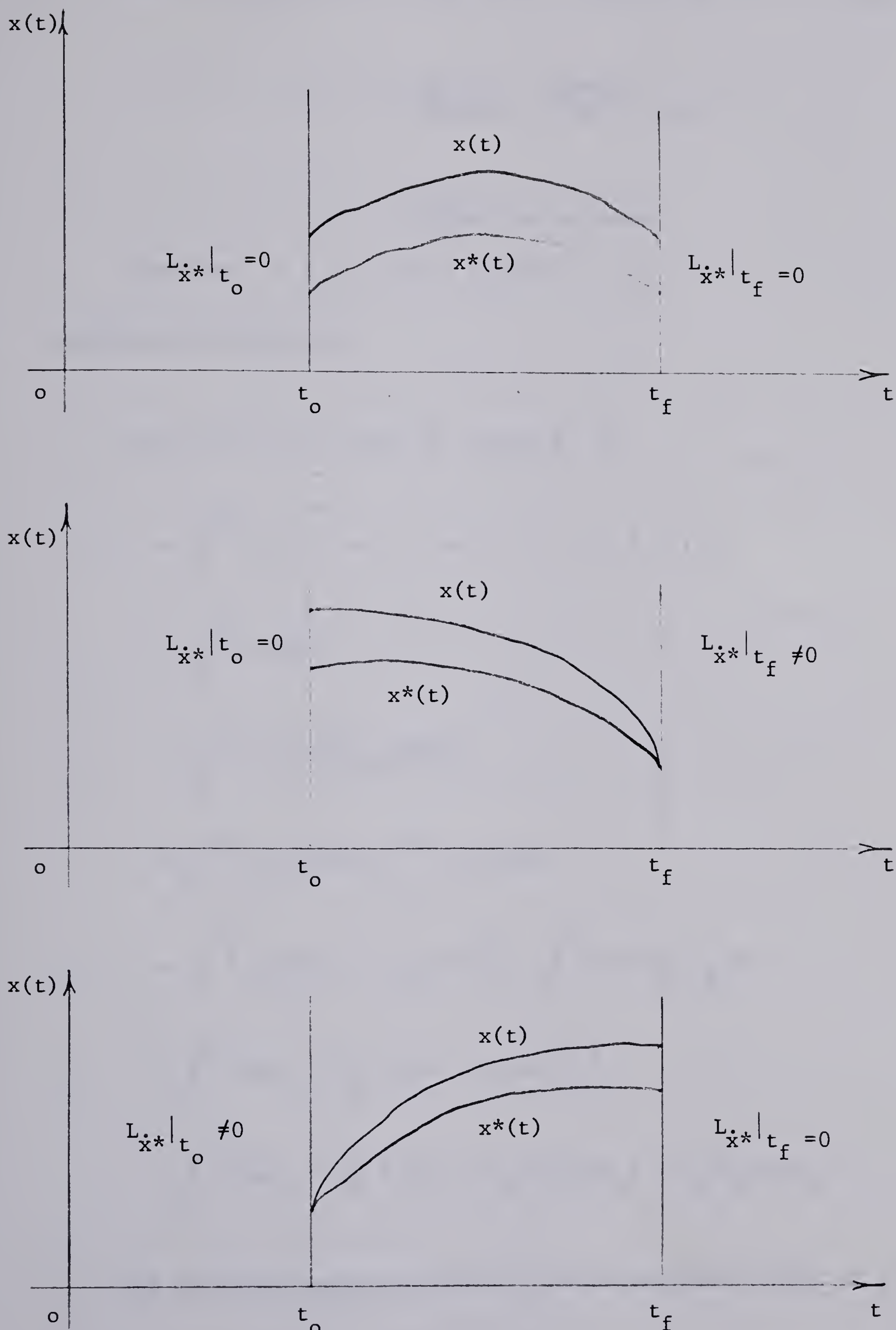

Fig. 2.2.1 Curves of Fixed and Free End Points

$$L(x + \varepsilon\eta,\ \dot{x} + \varepsilon\dot{\eta},\ t) = L(x,\ \dot{x},\ t) + \frac{\partial L}{\partial(x+\varepsilon\eta)} \frac{d}{d\varepsilon} (x + \varepsilon\eta)\Big|_{\varepsilon=o} \cdot \varepsilon$$

$$+ \frac{\partial L}{\partial(\dot{x}+\varepsilon\dot{\eta})} \frac{\partial(\dot{x}+\varepsilon\dot{\eta})}{\partial\varepsilon}\Big|_{\varepsilon=0} \cdot \varepsilon$$

+ higher order items.

Therefore, $\delta L = L_x \cdot (\delta x) + L_{\dot{x}} (\delta\dot{x})$

<u>b. First Variation of J</u>

$$\delta J = J(x + \varepsilon\eta,\ \dot{x} + \varepsilon\dot{\eta},\ t) - J(x,\ \dot{x},\ t)$$

$$= \int_{t_o}^{t_f} [L(x + \varepsilon\eta,\ \dot{x} + \varepsilon\dot{\eta},\ t) - L(x,\ \dot{x},\ t)]dt$$

$$= \int_{t_o}^{t_f} (\delta L)dt$$

$$= \int_{t_o}^{t_f} [L_x(\delta x) + L_{\dot{x}}(\delta\dot{x})]dt$$

$$= \int_{t_o}^{t_f} L_x(\delta x)dt + \int_{t_o}^{t_f} L_{\dot{x}}(\delta\dot{x})dt$$

$$= \int_{t_o}^{t_f} L_x(\delta x)dt + L_{\dot{x}}(\delta x)\Big|_{t_o}^{t_f} - \int_{t_o}^{t_f} (\delta x)(\frac{d}{dt} L_{\dot{x}})dt$$

$$= \int_{t_o}^{t_f} \delta x[L_x - \frac{d}{dt} L_{\dot{x}}]dt + L_{\dot{x}}(\delta x)\Big|_{t_o}^{t_f}$$

$$= \int_{t_o}^{t_f} \delta x[L_x - \frac{d}{dt} L_{\dot{x}}]dt + L_{\dot{x}}(t_f)\delta x(t_f) - L_{\dot{x}}(t_o)\delta x(t_o)$$

The necessary conditions for $x(t)$ to be extremal ($\delta J=0$) of J are:

(a) $L_x - \frac{d}{dt} L_{\dot{x}} = 0$

(b) $\frac{\partial L}{\partial \dot{x}} \Big|_{t_o}$ and/or $\frac{\partial L}{\partial \dot{x}} \Big|_{t_f} = 0$

2.3 Pontryagin's Minimum Principle

Pontryagin's minimum principle is an extension of the methods and results of variational calculus to the case of bounded control variables. If the set of admissible controls is unrestricted, the calculus of variations can be used to derive necessary conditions which characterize the optimal solution. When the admissible control set is bounded in some way, unrestricted variations in u(t) are not allowed.

The minimum principle can be considered as an extension or generalization of the calculus of variations to enable one to take account of systems whose input signals have constraints of certain types. Consider a system described by the state equation

$$\dot{x} = f(x, u, t) \tag{2.3.1}$$

with

$$x(t_o) = x_o$$

and

$$x(t_f) = x_f$$

$\underline{x}$ is the n-dim. state vector.

$\underline{u}$ is the m-dim. control vector.

The cost function to be extremized is

$$J = G[x(t_f), t_f] + \int_{t_o}^{t_f} L(x, u, t)dt \tag{2.3.2}$$

J, G and L are scalar functions of their arguments.

Consider the Lagrange problem in which G=0. Introduce a vector $\lambda(t)$ which is the costate vector and rewrite (2.3.2) as follows:

$$J = \int_{t_o}^{t_f} [L + \lambda^T(f - \dot{x})]dt \tag{2.3.3}$$

By replacing (2.3.2) by (2.3.3), we have converted a constrained minimization problem into an unconstrained minimization problem.

The variation δJ in J is:

$$\delta J = \int_{t_o}^{t_f} \{[(\frac{\partial L}{\partial x})^T + \lambda^T(\frac{\partial f}{\partial x}) + \dot{\lambda}^T] \ \delta x + [(\frac{\partial L}{\partial u})^T + \lambda^T(\frac{\partial f}{\partial u})]\delta u\}dt$$

Since $\delta x(t_f)$, δx and δu are arbitrary variations the requirement that δJ becomes zero yields the following conditions for $\underline{x}$, $\underline{u}$ to be optimal for $\underline{x}$, $\underline{u}$ to minimize J.

Setting the coefficient of $\delta\underline{x}$ equal to zero we obtain

$$(\frac{\partial L}{\partial x})^T + \lambda^T (\frac{\partial f}{\partial x}) + \dot{\lambda}^T = 0$$

or

$$\dot{\lambda} = -(\frac{\partial f}{\partial x})^T \lambda - \frac{\partial L}{\partial x} \tag{2.3.4}$$

Setting the coefficient of δu equal to zero we obtain

$$(\frac{\partial L}{\partial u})^T + \lambda^T(\frac{\partial f}{\partial u}) = 0 \tag{2.3.5}$$

A Hamiltonian function is defined as follows

$$H(x, \lambda, u, t) = L(x, u, t) + \lambda^T f(x, u, t) \qquad (2.3.6)$$

Differentiating (2.3.6) with respect to $\underline{\lambda}$

$$\frac{\partial H}{\partial \lambda} = f = \dot{x}$$

or

$$\dot{x} = \frac{\partial H}{\partial \lambda}$$

Differentiating (2.3.6) with respect to $\underline{x}$

$$\frac{\partial H}{\partial x} = \frac{\partial L}{\partial x} + \left(\frac{\partial f}{\partial x}\right)^T \lambda \qquad (2.3.7)$$

Comparing (2.3.4) and (2.3.7), we obtain

$$\dot{\lambda} = -\frac{\partial H}{\partial x}$$

Differentiating (2.3.6) with respect to $\underline{u}$

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \left(\frac{\partial f}{\partial u}\right)^T \lambda \qquad (2.3.8)$$

Comparing (2.3.5) and (2.3.8). We obtain

$$\frac{\partial H}{\partial u} = 0$$

2.3.1 Continuous Optimal Control Problem

Free end point case

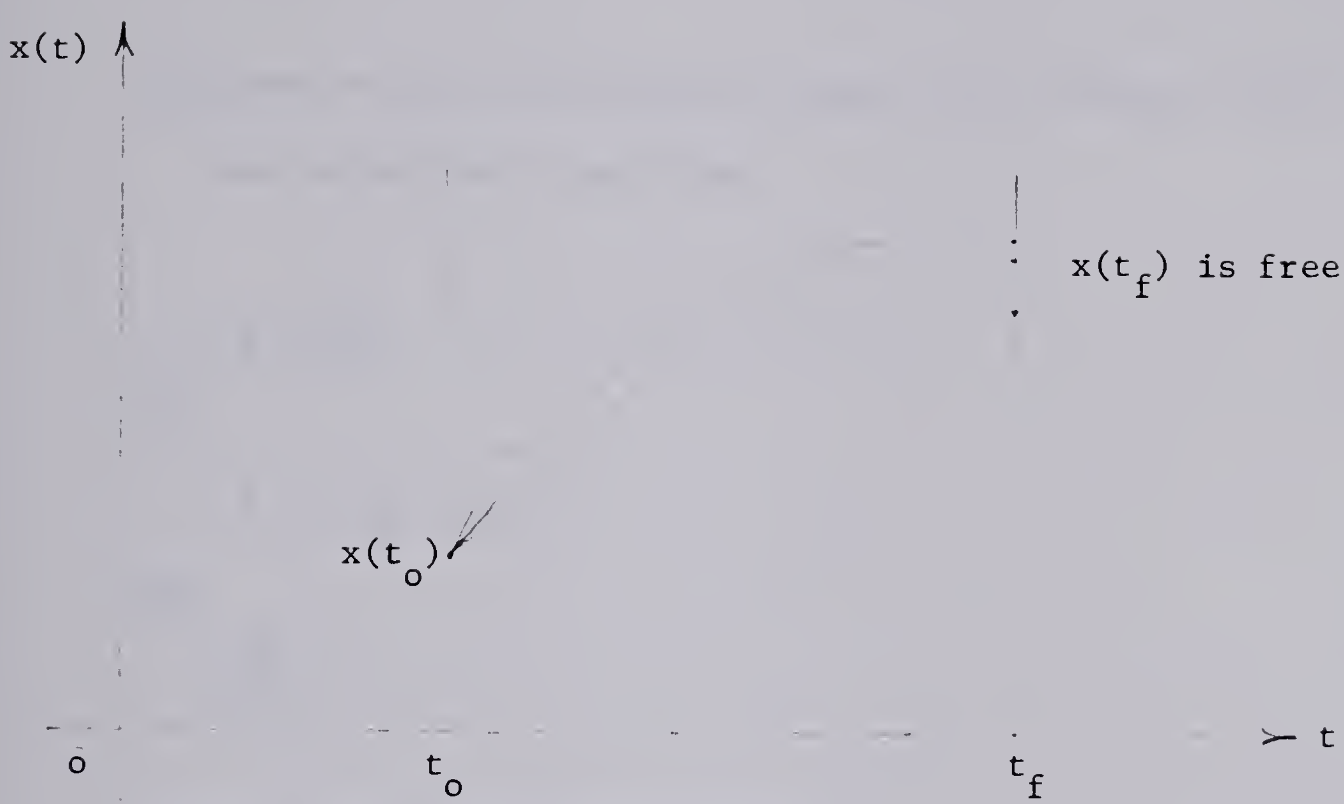

Fig. 2.3.1 Free End Point Case

Let u(t) be an admissible control and x(t) be the corresponding trajectory of the system described by

$$\dot{x}(t) = f(x, u, t)$$

Let $x(t_o) = x_o$, t_o and t_f be specified while $x(t_f)$ is free as shown in Fig. 2.3.1.

The necessary condition (a) and sufficient condition (b) for u(t) to be the optimal control which takes x(t) from x_o to some state $\underline{x}_f$ at t_f while minimizing the function J

$$J = \int_{t_o}^{t_f} [x(t)^T Q\, x(t) + u(t)^T R\, u(t)]dt$$

are:

(a) There exists a function or vector $\lambda(t)$ such that $x(t)$ and $\lambda(t)$ are solutions of equations

$$\dot{x} = \frac{\partial H}{\partial \lambda}$$

and

$$\dot{\lambda} = - \frac{\partial H}{\partial x}$$

and

$$\frac{\partial H}{\partial u} = 0$$

subject to the boundary conditions

$$x(t_o) = x_o$$

$$\lambda(t_f) = \lambda_f$$

(b) The matrix Q must be positive definite or positive semidefinite for t in $[t_o, t_f]$ in order to establish the sufficient condition for a minimum. The matrix R must also be positive definite for t in $[t_o, t_f]$.

The functional $H(x, \lambda, u, t)$ has a minimum if $\frac{\partial^2 H}{\partial u^2} > 0$ and $\frac{\partial H}{\partial u} = 0$.

2.4 Direct Computational Method

2.4.1 Gradient Techniques

Optimization methods can be divided into direct and indirect methods. The first one comprises various gradient methods whereby

the gradient of the cost functional with respect to the control function is used to successively estimate the optimal control. The second one is the quasilinearization method which involves two point boundary value problem by a sequence of solutions of a system of linear differential equations. This is denoted as a direct method of computation of optimal control.

Consider a point x for which H(x) is not a relative minimum. The Taylor series expansion is known:

$$H(x + h) = H(x) + hH'(x) + R_2$$

where the remainder terms are R_2.

Choose $h = -\alpha H'(x)$, $\alpha > o$, so that:

$$H(x + h) = H(x) - \alpha[H'(x)]^2 + R_2$$

Now, if α is sufficiently small such that

$|\alpha H'(x)| < \varepsilon$, then

$$|R_2| < \alpha[H'(x)]^2$$

and

$$H(x + h) < H(x).$$

This clearly indicates therefore that if we start with an

arbitrary initial guess x^o for x^*, where $H(x^*)$ is a relative minimum and update each successive estimate x^i according to the iterative algorithm:

$$x^{i+1} = x^i - \alpha H'(x^i) \quad i=1,2,3,\ldots \tag{2.4.1.1}$$

then

$$H(x^{i+1}) < H(x^i)$$

and x^i will eventually converge to x^*. The vector $H'(x)$ represents the gradient or the direction of steepest descent. Equation (2.4.1.1) is the gradient algorithm.

2.4.2 Direct Computation of Optimal Control

We assume that there are no inequality constraints, that the initial and terminal times are fixed, and that the initial state is fixed and the terminal state is unspecified.

Thus we wish to minimize

$$J = G[x(t_f),\ t_f] + \int_{t_o}^{t_f} L[x(t),\ u(t),\ t]dt \tag{2.4.2.1}$$

for the system

$$\dot{x} = f(x,\ u,\ t), \quad x(t_o) = x_o \tag{2.4.2.2}$$

We define the Hamiltonian

$$H(x,\ \lambda,\ u,\ t) = L(x,\ u,\ t) + \lambda^T f(x,\ u,\ t) \tag{2.4.2.3}$$

and then set

$$\frac{\partial H}{\partial x} = - \dot{\lambda} = \frac{\partial L(x,u,t)}{\partial x} + \lambda(t) \left[\frac{\partial f^T(x,u,t)}{\partial x}\right]$$

and then set

$$- \dot{\lambda} = \frac{\partial L(x,u,t)}{\partial x} + \lambda(t) \left[\frac{\partial f^T(x,u,t)}{\partial x}\right] \tag{2.4.2.4}$$

with the terminal condition

$$\lambda(t_f) = \frac{\partial G[x(t_f),t_f]}{\partial x(t_f)} \tag{2.4.2.5}$$

Also, we minimize the Hamiltonian with respect to a choice of u. For the case where any control is admissible, we use

$$\frac{\partial H}{\partial u} = 0 = \frac{\partial L(x,u,t)}{\partial u} + \lambda(t) \frac{\partial f^T(x,u,t)}{\partial u} \tag{2.4.2.6}$$

We shall now guess a solution for the optimal control, and therefore, we will not obtain $\frac{\partial H}{\partial u} = 0$. We will solve the differential system equality constraint (2.4.2.2) forwards in time from t_o to t_f with the assumed value of u and will also solve the adjoint equations (2.4.2.4) backwards in time from t_f to t_o with the terminal conditions (2.4.2.5).

Thus, the incremental first-order change in the cost function (2.4.2.1) becomes for a control differing by an amount $\delta u(t)$ from $u(t)$

$$\delta J = \int_{t_o}^{t_f} \left[\frac{\partial H(x,\lambda,u,t)}{\partial u(t)}\right]^T \delta u(t)dt \qquad (2.4.2.7)$$

If we wish to make the largest change in δJ, we would calculate the gradient $\frac{\partial H}{\partial u}$ and then make δu directed opposite to the gradient

$$\delta u(t) = - \alpha \frac{\partial H(x,\lambda,u,t)}{\partial u(t)} \qquad (2.4.2.8)$$

The change is such as to make J smaller if $\alpha > 0$, which is desirable since we wish to minimize J. To start the procedure, we assume that we have some nonoptimal control $u^i(t)$, we determine $x^i(t)$ by solving (2.4.2.2) and $\lambda(t)$ by solving (2.4.2.4) with the terminal condition.

The predicted change in J, δJ^i, may if desired be calculated from (2.4.2.7). A new trial value of $\underline{u}$

$$u^{i+1}(t) = u^i(t) + \delta u^i(t)$$

is then found and the procedure repeated until either the control or the cost function does not change significantly from iteration to iteration.

CHAPTER III

MODELLING OF THE SYNCHRONOUS MACHINE

3.1 Park's Transformation

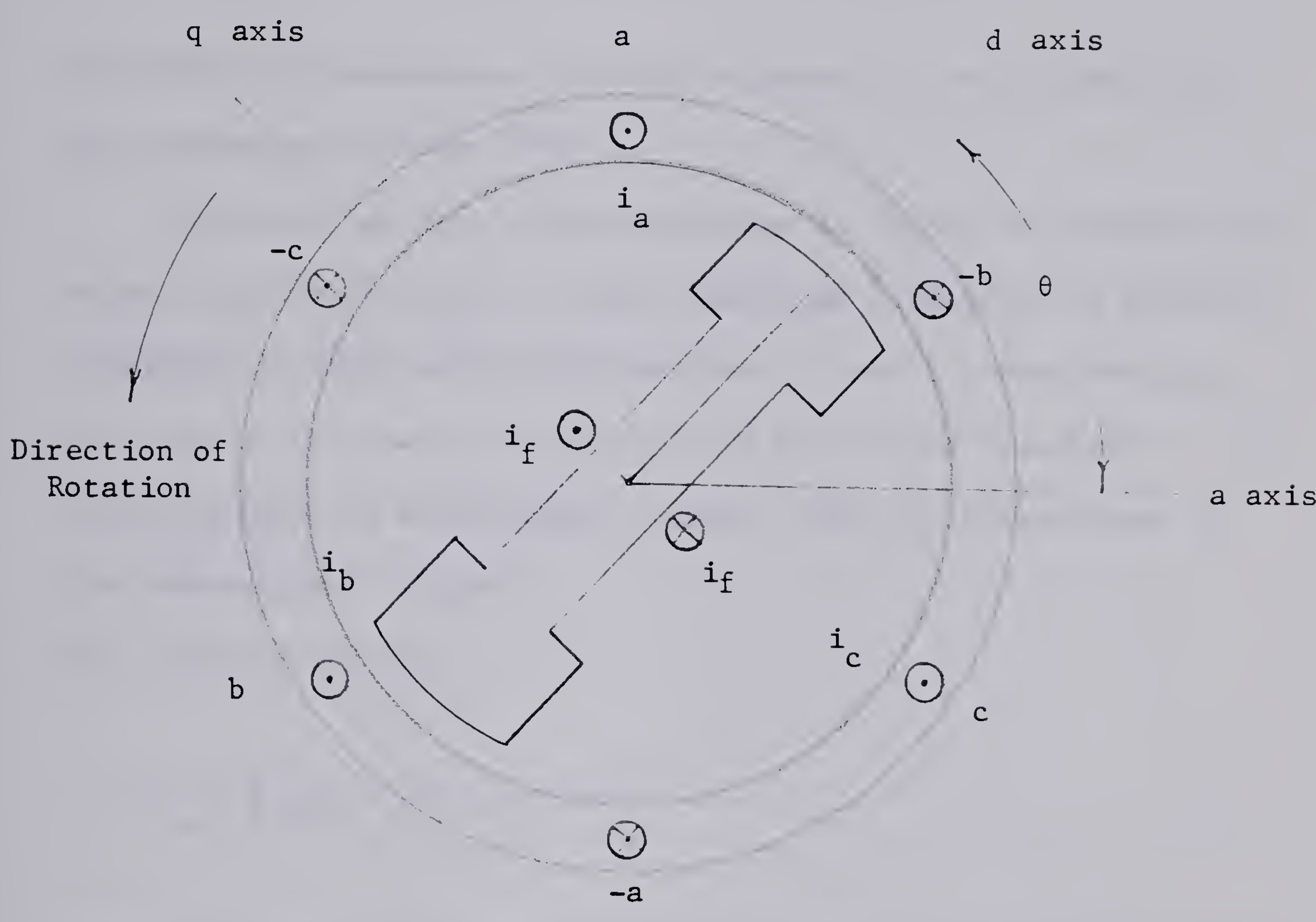

Fig. 3.1 Pictorial Representation of a Synchronous Machine

As shown in Fig. 3.1, the d axis of the rotor is defined at some instant of time to be at angle θ rad with respect to a fixed reference position. Let the stator phase current i_a, i_b and i_c be the currents entering the generator terminals. If these currents are projected along the d and q axis of the rotor, the relations are obtained as:

$$i_d = \frac{2}{3}\,[i_a \cos\theta + i_b \cos(\theta - 120°) + i_c \cos(\theta + 120°)]$$

$$i_q = \frac{2}{3}\,[i_a \sin\theta + i_b \sin(\theta - 120°) + i_c \sin(\theta + 120°)]$$

Note that for convenience the axis of phase "a" was chosen to be the reference position [15].

The effect of Park's transformation is simply to transform all stator quantities from a, b, and c into new variables the frame of reference of which moves with the rotor. Park's transformation uses two of the new variables as the d and q axes components. The third variable is a stationary current, which is proportional to the zero-sequence current.

This is given by [15]

$$i_o = \frac{1}{3}\,(i_a + i_b + i_c)$$

where i_o produces no space fundamental field in the air gap. Thus, by definition,

$$\begin{bmatrix} i_o \\ i_d \\ i_q \end{bmatrix} = P \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

where the Park's transformation P is defined as [18]:

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \cos\theta & \cos(\theta-120°) & \cos(\theta+120°) \\ \sin\theta & \sin(\theta-120°) & \sin(\theta+120°) \end{bmatrix} \qquad (3.1.1)$$

If the transformation (3.1.1) is unique, an inverse transformation also exists wherein one may write

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = P^{-1} \begin{bmatrix} i_o \\ i_d \\ i_q \end{bmatrix}$$

where the inverse may be computed to be

$$P^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \sqrt{\frac{1}{2}} & \cos\theta & \sin\theta \\ \sqrt{\frac{1}{2}} & \cos(\theta-120°) & \sin(\theta-120°) \\ \sqrt{\frac{1}{2}} & \cos(\theta+120°) & \sin(\theta+120°) \end{bmatrix}$$

Consider the expressions for the flux linking the coils shown in Fig. 3.1. These are given in matrix form in equation (3.1.2). In this equation,

$$
\begin{array}{l}
\text{stator} \\
\\
\\
\text{rotor}
\end{array}
\left[\begin{array}{c}
\lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_F \\ \lambda_D \\ \lambda_Q
\end{array}\right]
=
\left[\begin{array}{ccc|ccc}
L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\
L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\
L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ \hline
L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\
L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\
L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ}
\end{array}\right]
\left[\begin{array}{c}
i_a \\ i_b \\ i_c \\ \hline i_F \\ i_D \\ i_Q
\end{array}\right]
\begin{array}{l}
\text{stator} \\
\\
\\
\text{rotor}
\end{array}
\tag{3.1.2}
$$

3.1.1 Self Inductance of Stator

The phase-winding self-inductances are given by:

$$L_{aa} = A_o + A_2 \cos 2\theta$$

$$L_{bb} = A_o + A_2 \cos 2(\theta - 120°)$$

$$L_{cc} = A_o + A_2 \cos 2(\theta + 120°)$$

where A_o and A_2 are constants.

3.1.2 Mutual Inductances of Stator

The phase-to-phase mutual inductance are functions of θ but are symmetric.

$$L_{ab} = L_{ba} = - B_o - B_2 \cos 2(\theta + 30°)$$

$$L_{bc} = L_{cb} = - B_o - B_2 \cos 2(\theta - 90°)$$

$$L_{ca} = L_{ac} = - B_o - B_2 \cos 2(\theta + 150°)$$

where B_o and B_2 are constants.

3.1.3 Self-Inductances of Rotor

Since saturation and slot effect are neglected, all rotor self-inductances are constants.

$$L_{FF} = L_F$$

$$L_{DD} = L_D$$

$$L_{QQ} = L_Q$$

3.1.4 Mutual Inductances of Rotor

The mutual inductance between windings F and D is constant and does not vary with θ. The coefficient of coupling between the d and q axes is zero and all pairs of windings with 90° displacement have zero mutual inductance. Thus,

$$L_{FD} = L_{DF} = M_R$$

$$L_{FQ} = L_{QF} = 0$$

$$L_{DQ} = L_{QD} = 0$$

3.1.5 Mutual Inductances Between Stator and Rotor

All of these are functions of the rotor angle θ. From the phase windings to the field winding,

$$L_{aF} = L_{Fa} = C_1 \cos\theta$$

$$L_{bF} = L_{Fb} = C_1 \cos(\theta - 120°)$$

$$L_{cF} = L_{Fc} = C_1 \cos(\theta + 120°)$$

In what follows the damper windings are neglected, however, the effect due to these windings is taken into account by introducing a damping factor K_d in the dynamical equation. See equation (3.3.1).

3.2 Flux Linkage Equation

The sinusoidal flux wave exists in the air gap and it has a magnitude and angle in terms of this. One obtains more insight into the constants of the machine when one resolves this flux into the components q, d on the rotor and one can write the voltage E_a in terms of these components as follows [26]:

$$E_a = p(\psi_{md}\cos\theta + \psi_{mq}\sin\theta) + (R_a + L_1 p)i_a - L_m p\ i_b - L_m p\ i_c$$

where p is $\frac{d}{dt}$

L_1 is the leakage inductance of coil "a"

L_m is the part of mutual inductance between two armature coils due to flux that does not cross air gap.

Using $i_a + i_b + i_c = 3i_o$ and i_a in terms of i_d, i_q, one obtains,

$$E_a = p[(\psi_{md} + L_a i_d) \cos\theta + (\psi_{mq} + L_a i_q) \sin\theta]$$

$$+ L_o p i_o + R_a i_a$$

$$= p\,[\psi_d \cos\theta + \psi_q \sin\theta] + L_o i_o + R_a i_a$$

where

$$L_a = L_1 + L_m$$

$$L_o = L_a - 3L_m$$

$$\psi_d = \psi_{md} + L_a i_d$$

$$\psi_q = \psi_{mq} + L_a i_q$$

ψ_d, ψ_q are total flux linkages with an armature coil located on the appropriate axis due to both the main air gap flux and the armature leakage flux. L_a is the effective leakage inductance of either of the axis coils. L_o is the inductance associated with the zero-sequence current.

If one substitutes for E_a, i_a in terms of E_d, E_q, E_o, i_d, i_q, i_o, one gets

$$(E_d - R_a i_d) \cos\theta + (E_q - R_a i_q) \sin\theta + (E_o - R_a i_o - L_o p\, i_o)$$

$$= \cos\theta(p\psi_d + \omega\,\psi_q) + \sin\theta(p\psi_q - \omega\,\psi_d)$$

where $p\theta = \omega$.

Equating coefficients of $\sin\theta$, $\cos\theta$ terms independently, one gets

$$
\begin{aligned}
E_d &= p\psi_d + \omega\psi_q + R_a i_d \\
E_q &= -\omega\psi_d + p\psi_q + R_a i_q \qquad (3.2.1) \\
E_o &= (R_a + L_o p) i_o
\end{aligned}
$$

Now one must relate the fluxes ψ_d, ψ_q to the current i_d, i_q, i_f and also define the flux ψ_f and also find the expression for V_f.

$$
\begin{aligned}
\psi_d &= L_{fd}\, i_f + L_d\, i_d \\
\psi_q &= L_q\, i_q \\
V_f &= (R_f + pL_{ff}) i_f + L_{fd} p\, i_d
\end{aligned}
$$

where

L_{fd} is mutual inductance between coil f and d

L_d is self inductance of coil d

L_q is self inductance of coil q

The above equations are easier to handle if one uses per unit (p.u.).

In this case

$$
\begin{aligned}
L_{fd} &= L_{md} \\
L_d &= L_{md} + L_a \\
L_{ff} &= L_{md} + L_f \\
L_q &= L_{mq} + L_a
\end{aligned}
$$

where

L_{md} is per unit mutual inductance on the d axis

L_{mq} is per unit mutual inductance on the q axis

L_a,L_f are leakage per unit inductances.

$$\psi_d = (L_{md} + L_a)i_d + L_{md}\, i_f$$
$$\psi_q = (L_{mq} + L_a)i_q \qquad (3.2.2)$$
$$V_f = L_{md}p\, i_d + [R_f + (L_{md} + L_f)p]i_f$$

Let $\psi_f = L_{md}\, i_f + (L_{md} + L_f)i_f$, then $V_f = R_f\, i_f + p\, \psi_f$. From equation (3.2.2), the flux linkage equation is obtained as

$$\frac{d}{dt}(\omega_o \psi_f) = \omega_o V_f - \omega_o R_f i_f \qquad (3.2.3)$$

3.3 Torque Equation

$$M_t = M_e + J\frac{d\omega}{dt} + K_d\, \omega \qquad (3.3.1)$$

where

M_t is the total instantaneous applied torque

M_e is the total instantaneous electrical torque

$K_d\omega$ is the damping torque

J is the moment of inertia.

M_e is positive if power is passing into the machine from outside at a positive speed. If the machine accelerates constantly from the rest to ω_o in one second, then

$$J = \frac{2H}{\omega_o}$$

where the inertia constant H is:

$$H = \frac{\text{stored energy at synchronous speed in KW-sec}}{\text{Rated KVA}}$$

Substituting $J = \frac{2H}{\omega_o}$ into equation (3.3.1),

$$M_t = M_e + \frac{2H}{\omega_o} p\,\omega + K_d\,\omega.$$

Multiplying (3.2.2) by ω_o and redefining the constants in terms of impedances instead of inductances, one obtains:

$$\omega_o \psi_d = x_d i_d + x_{md} i_f$$

$$\omega_o \psi_q = x_q i_q$$

$$\omega_o \psi_f = x_{md} i_d + x_{fd} i_f$$

where

$$x_d = \omega_o (L_{md} + L_a) = x_{md} + x_a$$

$$x_q = \omega_o (L_{mq} + L_a) = x_{mq} + x_a$$

$$x_{md} = \omega_o L_{md}$$

$$x_{fd} = \omega_o (L_{md} + L_f) = x_{md} + x_f$$

It can be shown that if the machine is connected to an infinite bus by a series impedance x_e, R_e then one can change

$$x_a = x_a' + x_e$$

$$R_a = R_a' + R_e$$

(The primed quantities are original machine parameters). Consider the machine connected directly to the infinite bus, where E_d and E_q

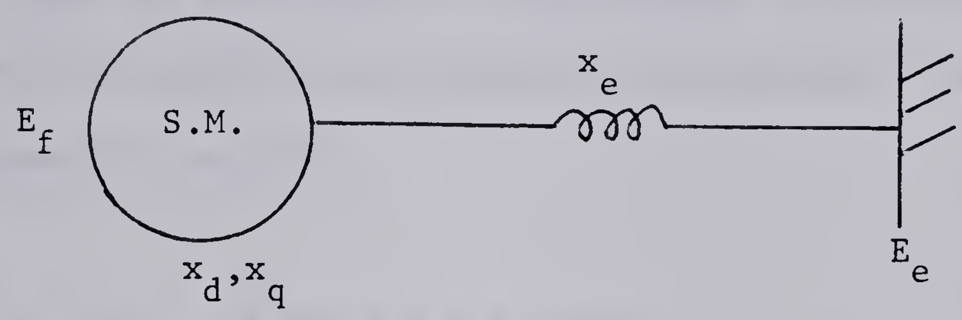

Fig. 3.3 Single-Line Diagram

are the components of the infinite bus voltage in the appropriate axis.

Now it is assumed in the voltage equations that as shown in Fig. 3.3

$$p\psi_d = 0$$
$$p\psi_q = 0$$
$$R_a = 0$$
$$R_e = 0$$
$$\omega = \omega_o .$$

Thus, it follows that

$$\begin{aligned}
\omega_o \psi_d &= x_d \, i_d + x_{md} \, i_f \\
\omega_o \psi_q &= x_q \, i_q \qquad\qquad (3.3.2) \\
\omega_o \psi_f &= x_{md} \, i_d + x_{fd} \, i_f
\end{aligned}$$

where

$$x_d = x'_d + x_e$$

$x_q = x'_q + x_e$ (The primed quantities are original machine parameters).

When the bus voltage is substituted in these equations, two dynamical equations are obtained as shown below. Let the voltage in phase "a" be [26]

$$E_a = E_{rms} \sqrt{2} \sin \omega_o t = E_m \sin \omega_o t$$

where

$$\sin \omega_o t = \sin(\theta + \delta) = \sin \theta \cos \delta + \cos \theta \sin \delta$$

then

$$E_a = E_m \sin \theta \cos \delta + E_m \cos \theta \sin \delta \tag{3.3.3}$$

Now from inverse Park's transformation, one has: [4]

$$E_a = \cos \theta \, E_d - \sin \theta \, E_q \tag{3.3.4}$$

Comparing (3.3.3) and (3.3.4), one has

$$E_d = E_m \sin \delta = \sqrt{2} \, E_{rms} \sin \delta$$

$$E_q = -E_m \cos \delta = -\sqrt{2} \, E_{rms} \cos \delta$$

Now one expresses the currents i_d, i_q, i_f and the fluxes

$\omega_o\psi_d$, $\omega_o\psi_q$ in terms of the bus E_{rms}, the angle δ and $\omega_o\psi_f$.

$$\omega_o\psi_d = \sqrt{2}\, E_{rms} \cos\delta$$

$$\omega_o\psi_q = \sqrt{2}\, E_{rms} \sin\delta$$

$$i_q = \frac{\sqrt{2}\, E_{rms} \sin\delta}{x_q}$$

From equation (3.3.2),

$$\begin{bmatrix} i_d \\ i_f \end{bmatrix} = \begin{bmatrix} x_d & x_{md} \\ x_{md} & x_{fd} \end{bmatrix}^{-1} \begin{bmatrix} \sqrt{2}\, E_{rms} \cos\delta \\ \omega_o\psi_f \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{x_{fd}}{x_d x_{fd} - x_{md}^2} & -\dfrac{x_{md}}{x_d x_{fd} - x_{md}^2} \\[2ex] -\dfrac{x_{md}}{x_d x_{fd} - x_{md}^2} & \dfrac{x_d}{x_d x_{fd} - x_{md}^2} \end{bmatrix} \begin{bmatrix} \sqrt{2}\, E_{rms} \cos\delta \\[2ex] \omega_o\psi_f \end{bmatrix}$$

Therefore

$$i_d = \frac{\sqrt{2}\, E_{rms}}{x_d - \dfrac{x_{md}^2}{x_{fd}}} \cos\delta - \frac{x_{md}}{x_{fd}} \left(\frac{1}{x_d - \dfrac{x_{md}^2}{x_{fd}}} \right) \omega_o\psi_f$$

$$i_f = \frac{-x_{md}}{x_{fd}} \left(\frac{1}{x_d - \dfrac{x_{md}^2}{x_{fd}}} \right) \sqrt{2}\, E_{rms} \cos\delta + \frac{x_d}{x_{fd}} \left(\frac{1}{x_d - \dfrac{x_{md}^2}{x_{fd}}} \right) \omega_o\psi_f$$

$$i_q = \frac{\sqrt{2}\, E_{rms} \sin\delta}{x_q} \tag{3.3.5}$$

$$\omega_o \psi_d = \sqrt{2}\, E_{rms} \cos \delta$$

$$\omega_o \psi_q = \sqrt{2}\, E_{rms} \sin \delta$$

Substituting (3.3.5) into $M_e = \frac{1}{2} (\omega_o \psi_d\, i_q - \omega_o \psi_q\, i_d)$, it becomes

$$M_e = S_4 \sin \delta \cos \delta + S_5\, \omega_o \psi_f \sin \delta \tag{3.3.6}$$

where

$$S_4 = \frac{- E_{rms}^2 (x_q - x_d + \frac{x_{md}^2}{x_{fd}})}{x_q \, (x_d - \frac{x_{md}^2}{x_{fd}})}$$

$$S_5 = \frac{E_{rms}\, x_{md}}{\sqrt{2}\, x_{fd}} \left(\frac{1}{x_d - \frac{x_{md}^2}{x_{fd}}}\right)$$

Now from the definition of δ, one has $\theta = \omega_o t + \delta$

$$\ddot{\theta} = \ddot{\delta} = \dot{\omega}$$

where ω_o is the synchronous speed

δ is the rotor angle with respect to a synchronous reference.

Mechanically one has

$$M \frac{d^2\theta}{dt^2} + K_d \frac{d\theta}{dt} = M_t' - M_e$$

where θ is the rotor position

M is the moment of inertia

K_d is the damping

M'_t is the mechanical torque of prime mover

M_e is the electrical torque

This corresponds to the electrical phase angle of the internal voltage

$$M \frac{d^2\delta}{dt^2} + K_d \frac{d\delta}{dt} = M_t - M_e$$

where $M_t = M'_t - K_d\,\omega_o$ is the net prime mover torque.

So it becomes

$$\frac{d^2\delta}{dt^2} = \frac{1}{M} (M_t - M_e - K_d \frac{d\delta}{dt}). \qquad (3.3.7)$$

Substituting (3.3.6) into (3.3.7), it becomes:

$$\frac{d^2\delta}{dt^2} = \frac{1}{M} [M_t - S_4 \sin\delta \cos\delta - S_5(\omega_o\psi_f) \sin\delta - K_d \frac{d\delta}{dt}]$$

where

$$M = \frac{2H}{\omega_o} .$$

Substituting (3.3.5) into (3.2.3), it becomes:

$$\frac{d(\omega_o\psi_f)}{dt} = \omega_o V_f - A(\omega_o\psi_f) + C \cos\delta$$

where

$$A = \frac{x_d}{x_{fd}} \left(\frac{1}{x_d - \frac{x_{md}^2}{x_{fd}}} \right) \omega_o R_f$$

$$C = \frac{\sqrt{2}\, E_{rms}\, x_{md}\, \omega_o R_f}{x_{fd} \left(x_d - \frac{x_{md}^2}{x_{fd}}\right)}$$

CHAPTER IV

MATHEMATICAL MODEL OF TURBO-GENERATOR

4.1 Machine Equations in State Space

If one defines $\dot{\delta} = \eta$, one obtains the following third order system from the previous chapter.

$$\dot{\delta} = \eta$$

$$\dot{\eta} = \frac{u_1'}{M} - \frac{S_4}{M} \sin\delta \cos\delta - \frac{S_5}{M} x_3 \sin\delta - \frac{K_d}{M} x_2$$

$$\dot{x}_3 = u_2' - A x_3 + C \cos\delta \qquad (4.1.1)$$

Summarizing the assumptions used in the derivation of this third system;

1. Damper windings were not considered, instead a damping coefficient is introduced in the torque equation.
2. Transient response in the transmission line is neglected.
3. The resistances in the stator winding, the transformer and the transmission line are neglected.
4. The time derivative of the fluxes ψ_d, ψ_q is neglected, i.e. $\dot{\psi}_d = 0$, $\dot{\psi}_q = 0$.
5. ω is approximated to ω_o in the voltage equations.

Equations (4.1.1) represent a synchronous machine to a very good approximation [11]. When use is made of the transformations

$$x_1 = \sin\delta$$
$$x_2 = \eta \qquad (4.1.2)$$

the machine equations (4.1.1) become

$$\dot{x}_1 = x_2 x_4$$

$$\dot{x}_2 = \frac{u_1'}{M} - \frac{S_4}{M} x_1 x_4 - \frac{S_5}{M} x_1 x_3 - \frac{K_d}{M} x_2 \qquad (4.1.3)$$

$$\dot{x}_3 = u_2' - A\, x_3 + C\, x_4$$

where

$$x_4^2 = 1 - x_1^2 \ .$$

It is assumed that the disturbance is modelled by introducing a torque pulse in the right hand side of the torque equation of magnitude K which is increased in steps of 5% of steady state value M_t^s, and a duration τ (0.5 sec). Hence in this model, the values of the variables at $t = t_o$ are the steady state values.

Therefore, the machine equations (4.1.3) for this model are modified to

$$\dot{x}_1 = x_2\, x_4$$

$$\dot{x}_2 = \frac{1}{M} [u_1' - S_4\, x_1 x_4 - S_5\, x_3 x_1 - K_d x_2 + K[u(t-t_o) - u(t-\tau)]$$

$$\dot{x}_3 = u_2' - A\, x_3 + C\, x_4 \qquad (4.1.4)$$

$$\dot{x}_4 = - x_2 x_1$$

where

$u_1' = M_t$

$u_2' = \omega_o V_f$

Here the last equation in (4.1.3) which is algebraic is converted into a differential equation as in [2] in order that the equality constraints are unified in structure so that it is easier to find a unified approach for the implementation of a numerical solution of the system.

4.2 Modelling of the Turbine, the Governor and the Exciter

4.2.1 Steam Turbine

To develop the dynamic characteristic of a steam turbine, consideration must be given to the energies associated with the steam system and the mechanical rotating system. The normal controlling input for a turbine is the control value and the output is the mechanical power delivered on its output shaft.

Usually the turbine is solidly connected to its generator so the inertial effects of both units are considered together as additive. For this dynamic model, the effects of boiler pressure changes can be ignored so that the steam control valve can be considered as a power controller.

The steam turbines in an electric power system normally consist of two or more steam expansion stages with moisture-separator and reheat stages between them. The transfer function relating the power developed by a turbine stage to the steam flow of the stage is given by: [12]

$$\frac{P_{in}(k)}{P_s(k)} = \frac{1}{1 + T_b(k)s}$$

where

$P_{in}(k)$ is the power (per unit) developed by the k^{th} stage,

$P_s(k)$ is the flow of steam (per unit) into the k^{th} stage,

$T_b(k)$ is the characteristic time constant of the k^{th} stage.

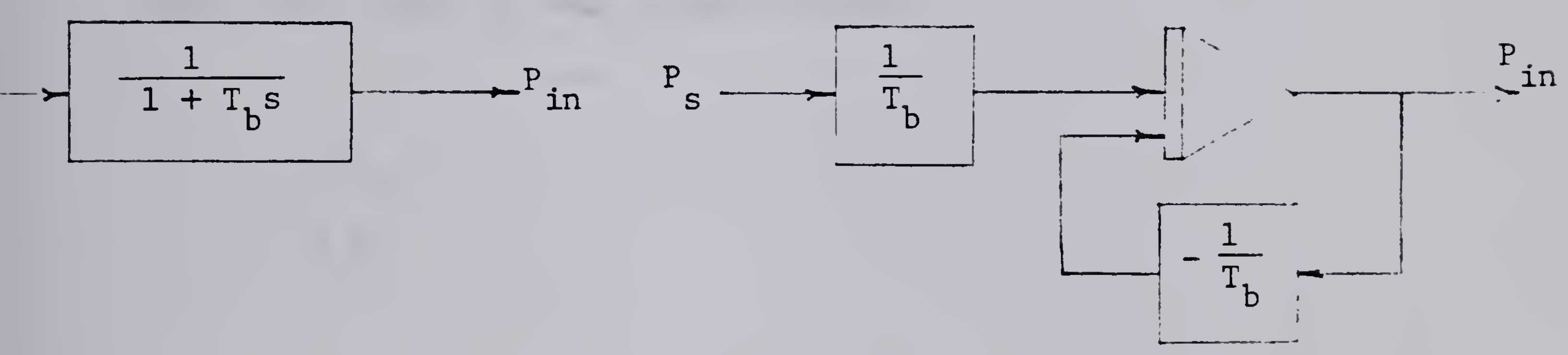

Fig. 4.2.1 Block Diagram of Steam Turbine

For this steam turbine the transfer function relating input valve movement to output mechanical power is:

$$\frac{P_{in}}{P_s} = \frac{1}{1 + T_b s} \tag{4.2.1.1}$$

Note that the transfer function is a linearized approximation based on conditions at a specific operating level.

The development of the differential equations for a typical double expansion turbine is shown in [12]. The same techniques

can be employed to develop transfer function (4.2.1.1) for this type of unit.

$$\dot{P}_{in} = \frac{1}{T_b} P_s - \frac{1}{T_b} P_{in} \qquad (4.2.1.2)$$

where

P_{in} is the mechanical power

P_s is the steam power

T_b is the time constant of the turbine

when use is made of the transformations

$$x_5 = P_{in}$$
$$x_7 = P_s$$

the turbine equation (4.2.1.2) becomes:

$$\dot{x}_5 = \frac{1}{T_b} [x_7 - x_5] \qquad (4.2.1.3)$$

4.2.2 Speed Governor

The speed governor used on the generator prime mover may be considered either as a speed controller or power controller depending on the mode of generator operation. If the electric generator is supplying an isolated load, the governor tends to operate as a speed controller; however if the generator is connected to a large system where the total generating capacity is much larger than that of the

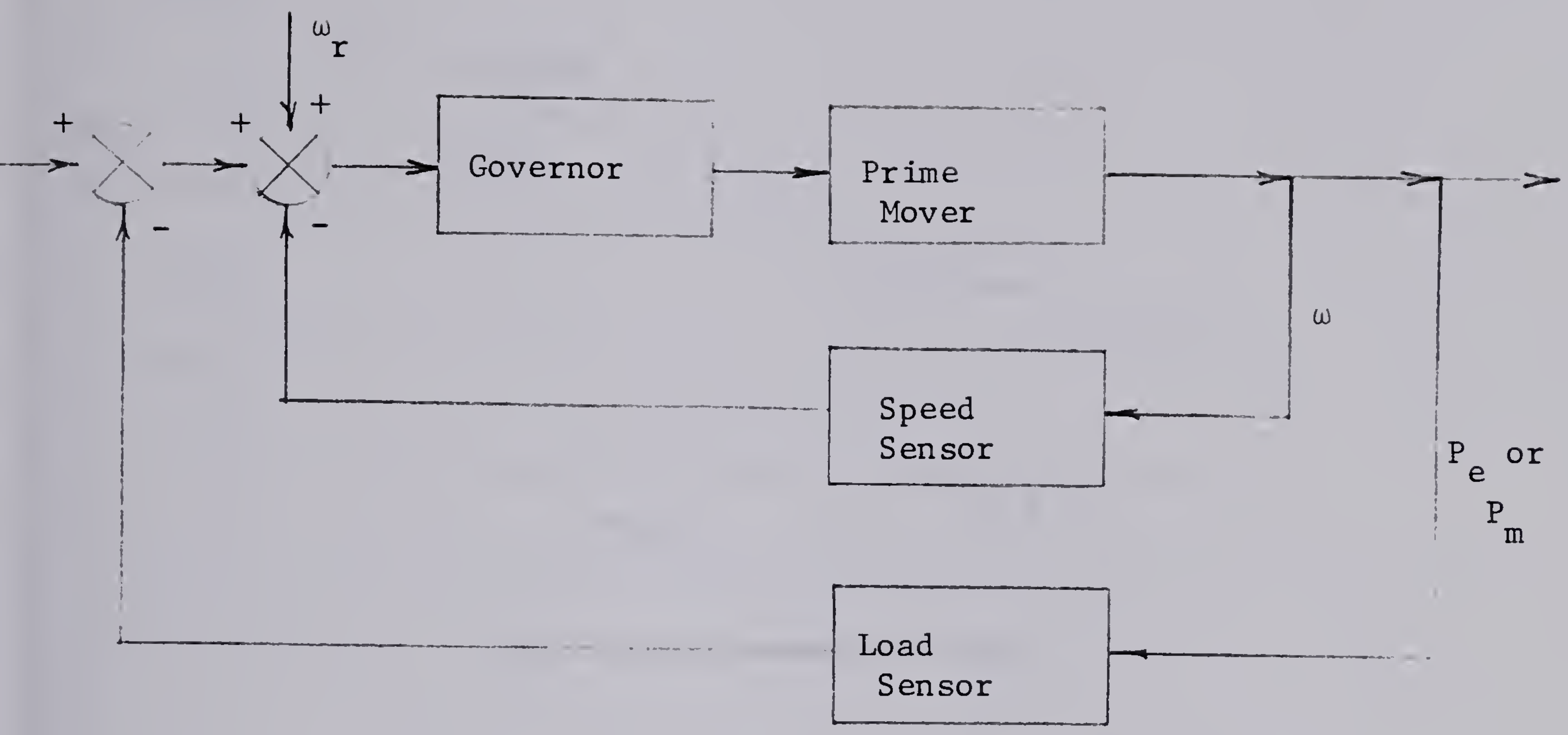

Fig. 4.2.2.1 Functional Elements of a Speed Governor

unit itself, the frequency cannot be appreciable altered by the individual unit and therefore the governor operates as a power controller. The general arrangement of the functional elements of the governor is shown in Fig. 4.2.2.1.

For speed and power control study purposes, both the steady-state gain and the transient performance of governor are of interest. To develop the transfer function of a typical speed governor, the steady-state droop characteristic has to be considered.

The steady-state performance of a governor depends on the value of the steady-state speed droop R. That is, under steady-state

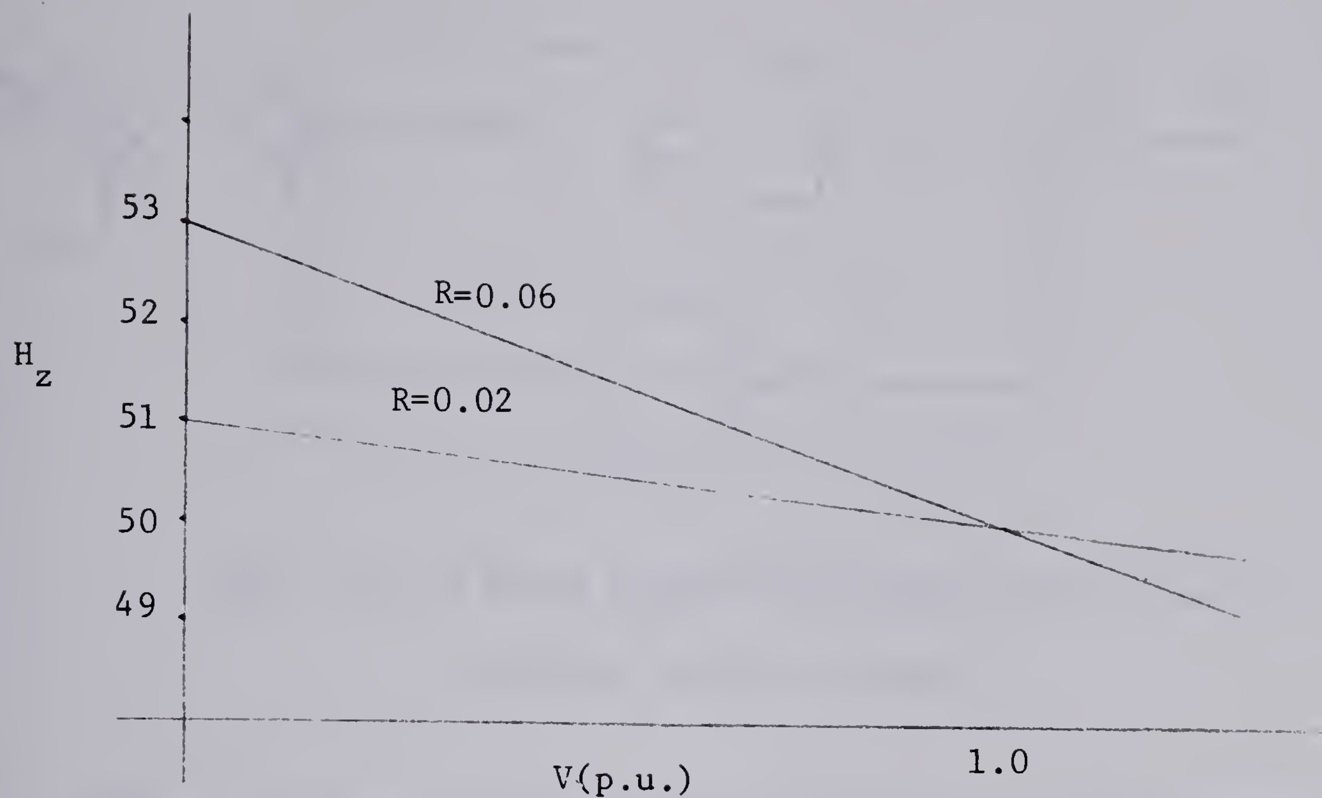

Fig. 4.2.2.2 Governor Droop

conditions

$$\Delta V = \frac{1}{R}\,\Delta\omega$$

where

ΔV is the amount of turbine inlet valve movement

R is the governor droop

$\frac{1}{R}$ is referred to as the steady-state gain

$\Delta\omega$ is the speed error.

Fig. 4.2.2.3 illustrates the most common transfer function for steam turbine governor relating speed change $\Delta\omega$ to steam control valve movement ΔV. With reference to the governor characteristic shown in Fig. 4.2.2.2, adjustment of R in Fig. 4.2.2.3 has the effect of adjusting the slope of the characteristic while variation of ω_r

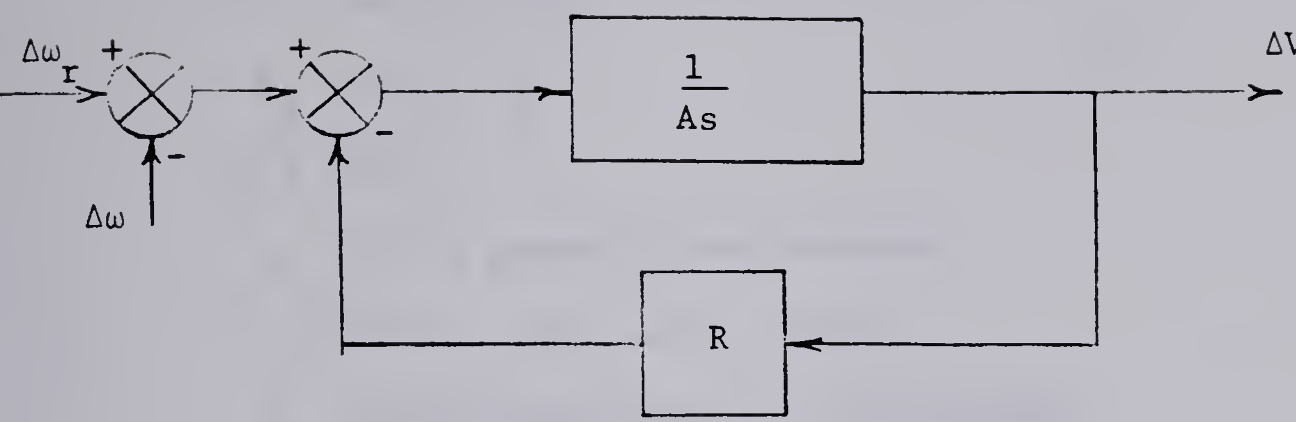

Fig. 4.2.2.3 Block Diagram and Transfer Function for a Steam Turbine Governor

adjusts the whole characteristic up and down while maintaining the same slope.

From Fig. 4.2.2.3, we obtain:

$$\frac{\Delta V}{\Delta\omega_r} = \frac{1/R}{1 + T_g s} \qquad (4.2.2.1)$$

where

T_g is the time constant of the governor

T_g is A/R

A is the governor constant

Substituting P_s, Y_o into (4.2.2.1) V, ω_r, the transfer function (4.2.2.1) becomes:

$$\frac{P_s}{Y_o} = \frac{G_2 G_3}{1 + T_g s} \qquad (4.2.2.2)$$

where

P_s is $G_3(V \cdot M_t)$

Y_o is $\omega_r \cdot M_t$

$1/R$ is G_2

G_2 is the governor gain constant

G_3 is the steam valve constant

T_g is the time constant of the governor

Y_o is the speeder gear setting.

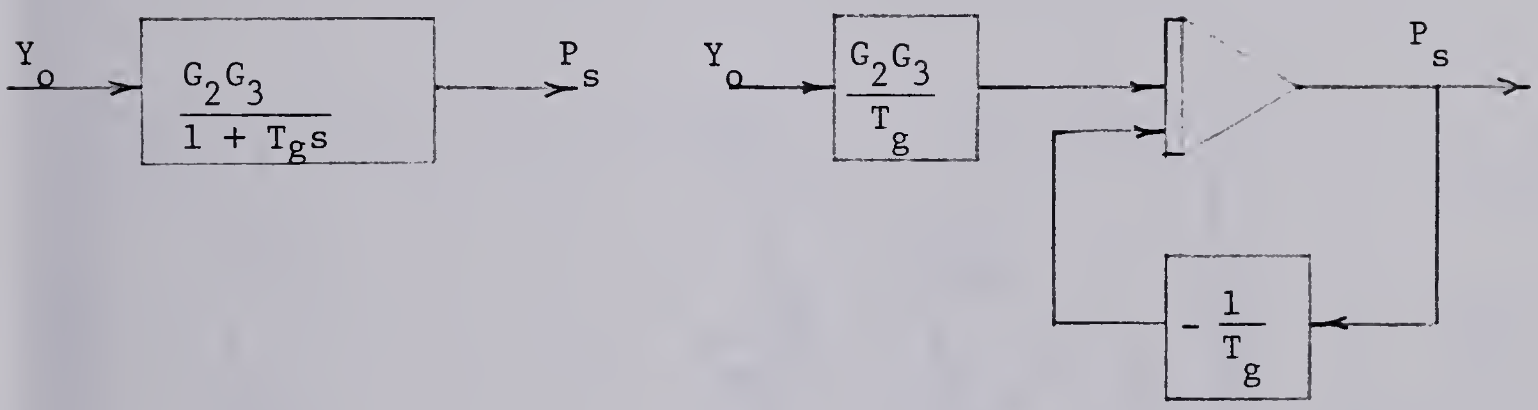

Fig. 4.2.2.4 Block Diagram of a Steam Turbine Governor

The transfer function (4.2.2.2) can be converted into the following differential equation.

$$\dot{P}_s = \frac{G_2G_3}{T_g} Y_o - \frac{1}{T_g} P_s \tag{4.2.2.3}$$

When use is made of the transformations

$x_7 = P_s$

$u_1 = Y_o$

the governor equation (4.2.2.3) becomes

$$\dot{x}_7 = \frac{1}{T_g} [G_3 G_3 u_1 - x_7] \tag{4.2.2.4}$$

4.2.3 Exciter

4.2.3.1 Automatic Voltage Regulator (A.V.R.)

In most modern generators the output voltage is controlled by automatic devices so as to remain at a constant prearranged value.

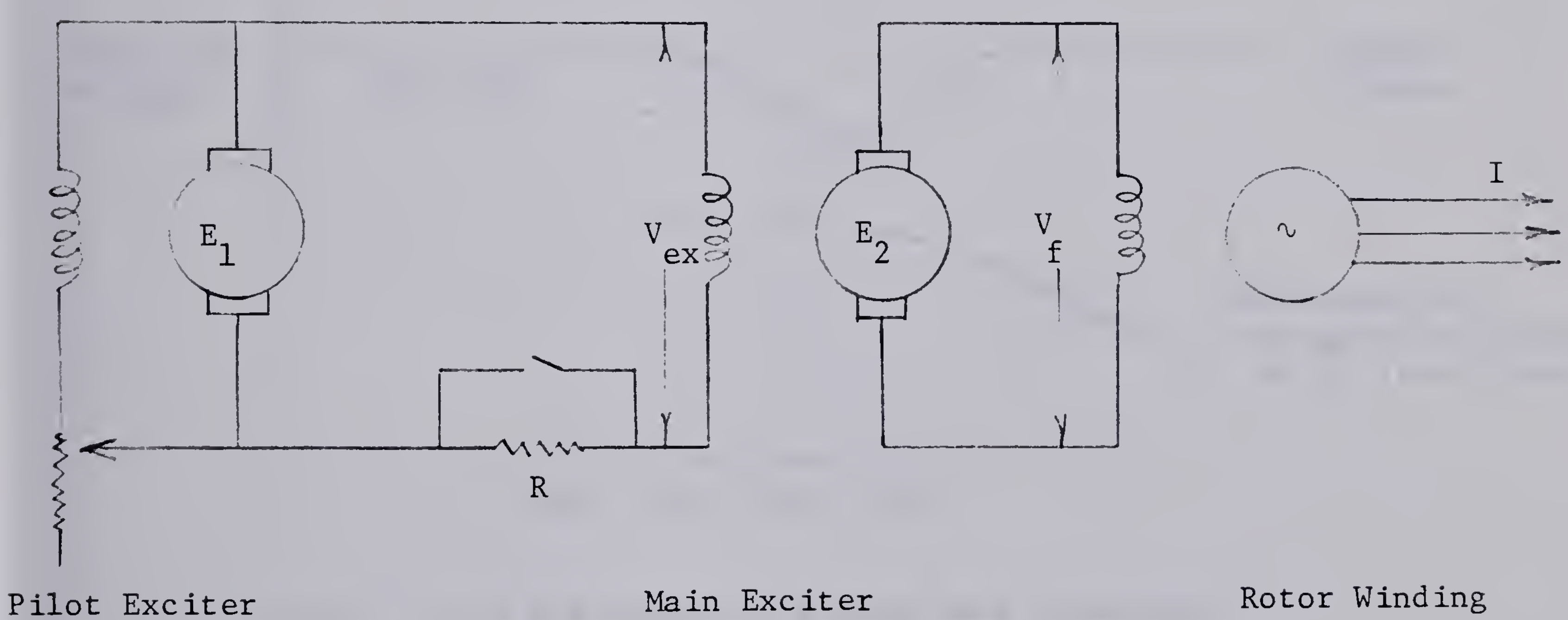

Fig. 4.2.3.1 Excitation Arrangements for a Synchronous Generator

The excitation arrangements for the rotor of a synchronous generator is shown in Fig. 4.2.3.1. There are two broad divisions

of automatic regulator both of which set out to control the output voltage of the synchronous generator by controlling the exciter field. In general the deviation of the terminal voltage from a prescribed value is passed to control circuits and thus the field current is varied.

One type can be broadly classed as electromechanical. Here a voltage proportional to the deviation voltage operates a solenoid assembly to vary the pressure exerted on a carbon-pile resistor in the exciter field, thus varying its resistance.

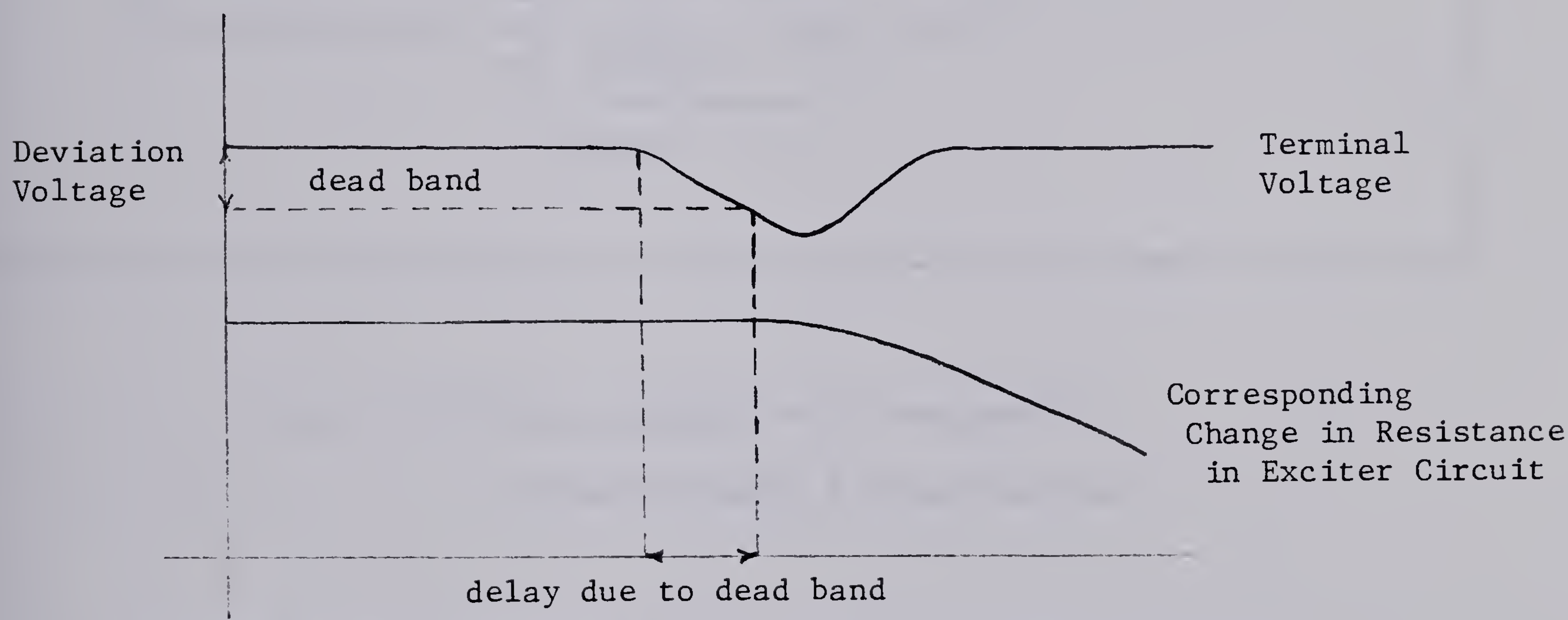

Fig. 4.2.3.2 The Effect of a Dead Band in Automatic Voltage Regulator

This type suffers from the disadvantages of being relatively slow acting and processing dead-bands, i.e. a certain deviation must occur before the mechanism operates as shown in Fig. 4.2.3.2.

The other main group of regulators is known as continuously acting and these are faster than the above and have no dead bands.

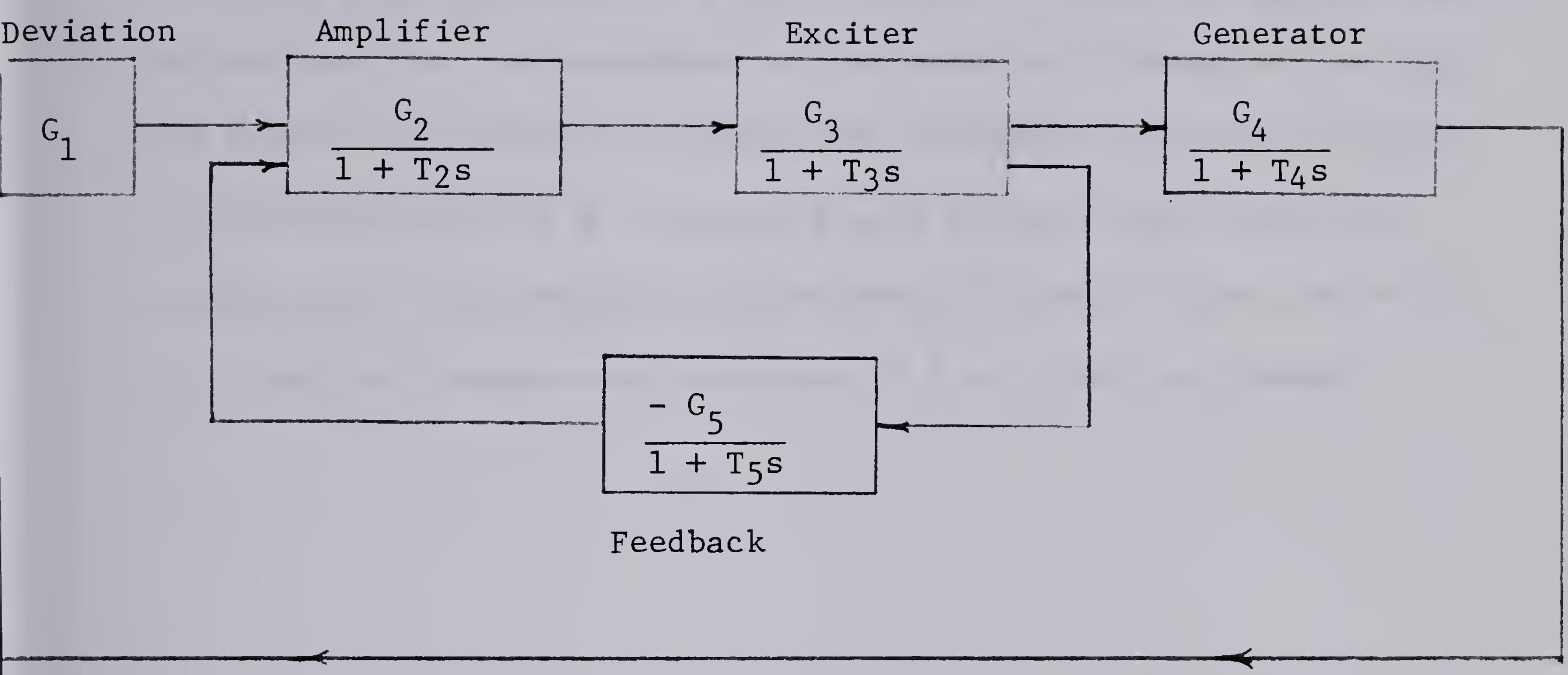

Fig. 4.2.3.3 Block Diagram of a Continuously Acting Automatic Voltage Regulator

A general block diagram of a typical control system is given in Fig. 4.2.3.3.

4.2.3.2 <u>The State Equation of Exciter</u>

In the present investigation, an exciter is assumed to be present, but there is no feedback applied to it from the machine terminal.

The terminal voltage can be represented only as a complicated function of the state variables δ and $(\psi_f\ \omega_o)$ and cannot easily be included in the dynamic equations of the system. The voltage V_{ex}

applied to the field winding of the exciter is assumed to be available for direct manipulation and is used as one of the control input variables.

The effect of the exciter is approximately represented by a single time constant and a suitable gain. There are various forms of exciters and the parameters of the model will depend on the type of exciter considered. For thyristor rectifier excitation supplied from transformers or a.c. exciters with silicon diode rectified output, the time constants vary between 0.05 and 0.3 sec. while for d.c. exciters larger time constants of 1 to 2 sec. are common.

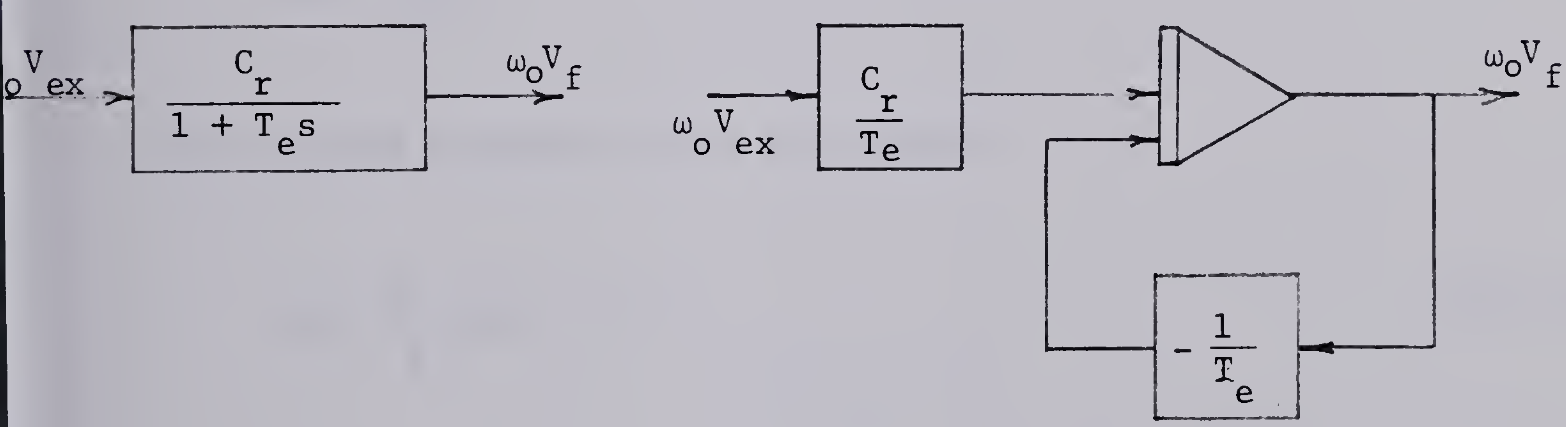

Fig. 4.2.3.4 Block Diagram of Exciter

The transfer function of the exciter is:

$$\omega_o V_f = \frac{C_r}{1 + T_e s} \omega_o V_{ex} \tag{4.2.3.1}$$

where

C_r is the exciter gain constant

T_e is the time constant of the exciter

$\omega_o V_{ex}$ is the exciter field voltage times the synchronous speed

$\omega_o V_f$ is the field voltage times the synchronous speed.

In the time domain equation (4.2.3.1) becomes:

$$\omega_o \dot{V}_f = \frac{1}{T_e} [C_r \ \omega_o V_{ex} - \omega_o V_f] \tag{4.2.3.2}$$

When use is made of the transformations

$$x_6 = \omega_o V_f$$

$$u_2 = \omega_o V_{ex}$$

the exciter equation (4.2.3.2) becomes:

$$\dot{x}_6 = \frac{1}{T_e} [C_r u_2 - x_6] \tag{4.2.3.3}$$

CHAPTER V

MATHEMATICAL SOLUTION OF THE MODEL

5.1 System Equations in State Space

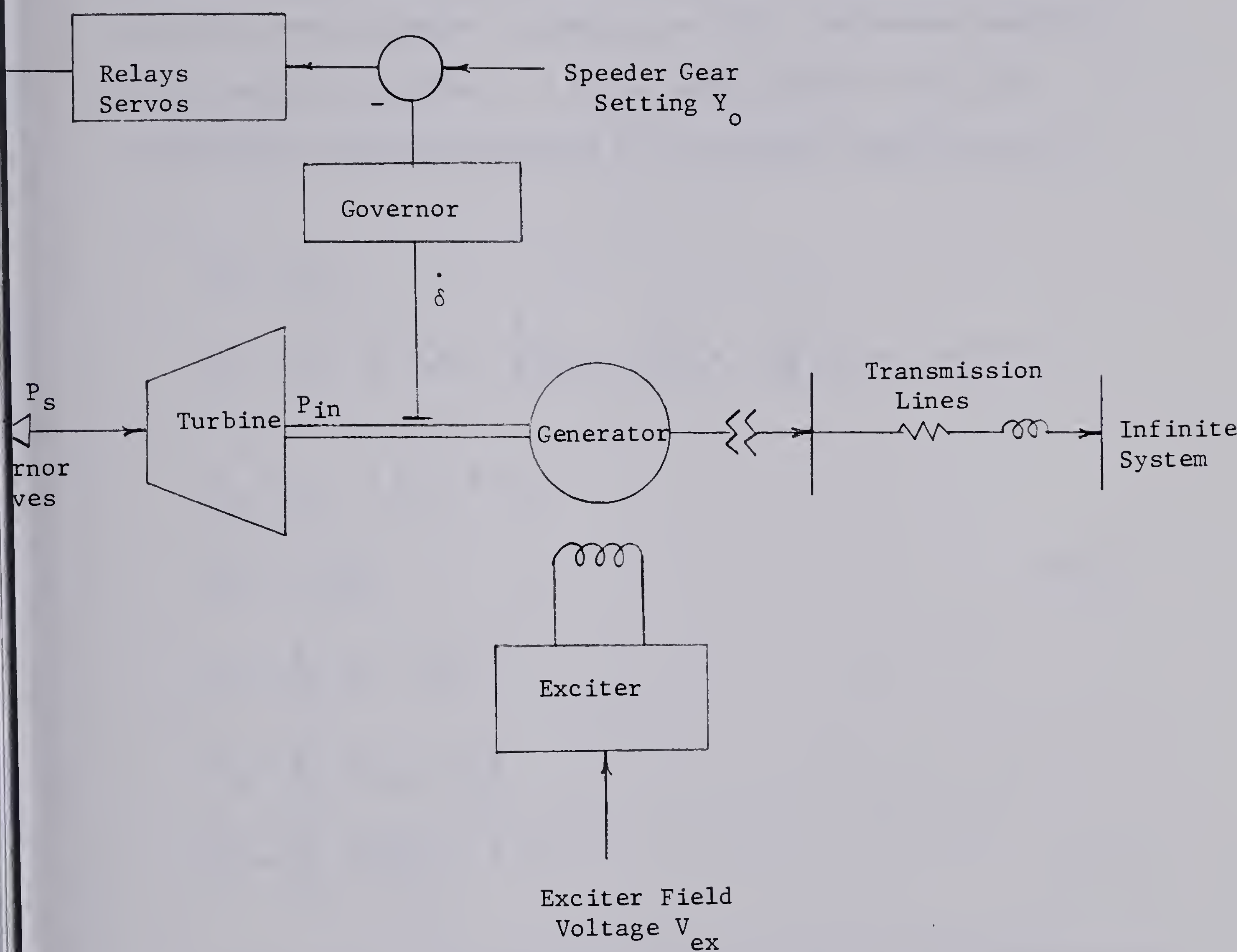

Fig. 5.1 Schematic Diagram of the System Investigated

The model used here is a specialized form of the mathematical model (4.1.4) derived in Chapter 3, since we review the mathematical

model in Chapter 4 and present its specialization for this chapter.

Inclusion of the transfer functions for the turbine, the governor and the exciter will add three more additional differential equations (4.2.1.3), (4.2.2.4), (4.2.3.3) to those in Section 4.2. Here the same procedure is used as in [12]. The system equations for a machine disturbed by a torque pulse together with the differential equation generated by the transfer functions are:

$$\begin{aligned}
\dot{x}_1 &= x_2 x_4 \\
\dot{x}_2 &= \frac{x_5}{M} - \frac{S_4}{M} x_1 x_4 - \frac{S_5}{M} x_3 x_1 - \frac{K_d}{M} x_2 + \frac{K}{M} [u(t-t_o)-u(t-\tau)] \\
\dot{x}_3 &= x_6 - A x_3 + C x_4 \\
\dot{x}_4 &= - x_2 x_1 \\
\dot{x}_5 &= \frac{1}{T_b} [x_7 - x_5] \\
\dot{x}_6 &= \frac{1}{T_e} [C_r u_2 - x_6] \\
\dot{x}_7 &= \frac{1}{T_g} [G_2 G_3 u_1 - x_7]
\end{aligned} \qquad (5.1.1)$$

Here x_1, x_2, x_3 and x_4 are the $\sin\delta$, derivative of torque angle, field flux and $\cos\delta$ respectively. x_5, x_6 and x_7 are the mechanical power, the field voltage times the synchronous speed and the steam power respectively. S_4, S_5, K_d, M, A, and C are the machine parameters derived in Chapter 3. T_b, T_g, and T_e are the time constants of the turbine, the governor and the exciter respectively, while G_2, G_3, and

C_r are the governor, the steam valve, and the exciter gain constants respectively.

The control signal is assumed to control directly the speeder gear setting of the governor mechanism and thereby the valve opening. One of the control input variables u_1 is assigned to Y_o which represents the speeder gear setting. The other control input variable u_2 is the voltage V_{ex} applied to the field of the exciter multiplied by ω_o for convenience as shown in Fig. 5.1.

Before use is made of the transformations

$$x_1 = \sin\delta$$
$$x_2 = \eta$$
$$x_3 = \omega_o\psi_f$$
$$x_4 = \cos\delta$$

the cost function to be minimized is:

$$J = \int_{t_o}^{t_f} [\alpha_1(\delta-\delta^s)^2 + \alpha_2\eta^2 + \alpha_3(x_3-x_3^s)^2 + \gamma_1(x_5-x_5^s)$$
$$+ \gamma_2(x_6-x_6^s)^2 + \gamma_3(x_7-x_7^s)^2 + \beta_1(u_1-u_1^s)^2 + \beta_2(u_2-u_2^s)^2]$$

where superscript s indicates the steady state value. The parameters α_1, α_2, α_3, γ_1, γ_2, γ_3, β_1 and β_2 are penalty factors for deviations from steady state values.

Due to the transformation (4.1.2) one should express $(\delta-\delta^s)^2$

in the cost function in terms of $(x_1-x_1^s)$ and then expand the resulting function and introduce additional pseudo-variables. On the other hand, one could alter the cost function slightly and if necessary increase the factor α, to compensate for this change as will be shown below.

A cost function with $(\delta-\delta^s)^2$ replaced by $(x_1-x_1^s)^2 + (x_4-x_4^s)^2$ is monotomically increasing in the range $(0,\pi)$ and approximates $(\delta-\delta^s)^2$ to a high degree of accuracy over the range $(0, \pi/2)$. This can be seen as follows:

$$(x_1-x_1^s)^2 + (x_4-x_4^s)^2 = 2[1 - \cos(\delta-\delta^s)]$$

If one now chooses α $(\alpha = 2.5\ \alpha_1)$ for the coefficient of $[(x_1-x_1^s)^2 + (x_4-x_4^s)^2]$ in the cost function , one will have the same penalty at π for both the old and new cost function.

It should be noticed that in this case small errors are penalized more heavily than before which is a beneficial feature of the modified cost function. Now the modified cost function becomes:

$$J = \int_{t_o}^{t_f} [\alpha[(x_1-x_1^s)^2 + (x_4-x_4^s)^2] + \alpha_2 x_2^2 + \alpha_3(x_3-x_3^s)^2$$

$$+ \gamma_1(x_5-x_5^s)^2 + \gamma_2(x_6-x_6^s)^2 + \gamma_3(x_7-x_7^s)^2 + \beta_1(u_1-u_1^s)^2$$

$$+ \beta_2(u_2-u_2^s)^2]dt \qquad (5.1.2)$$

The control is specialized to be, for this problem, of the form

$$u_1-u_1^s = a_1(x_1-x_1^s) + a_2(x_2-x_2^s) + a_3(x_3-x_3^s) + a_4(x_4-x_4^s)$$
$$u_2-u_2^s = b_1(x_1-x_1^s) + b_2(x_2-x_2^s) + b_3(x_3-x_3^s) + b_4(x_4-x_4^s) \tag{5.1.3}$$

where

a_1, a_2, a_3 and a_4 are constants

b_1, b_2, b_3, and b_4 are constants.

However the magnitude of these constants in general depends on the strength of the disturbance. These constants clearly satisfy

$$\dot{a}_1 = \dot{a}_2 = \dot{a}_3 = \dot{a}_4 = 0$$
$$\dot{b}_1 = \dot{b}_2 = \dot{b}_3 = \dot{b}_4 = 0 \tag{5.1.4}$$

5.2 Mathematical Solution of the Model

By substituting

$$\begin{aligned} y_1 &= x_1 - x_1^s \\ y_2 &= x_2 - x_2^s \\ y_3 &= x_3 - x_3^s \\ y_4 &= x_4 - x_4^s \\ y_5 &= x_5 - x_5^s \\ y_6 &= x_6 - x_6^s \\ y_7 &= x_7 - x_7^s \\ v_1 &= u_1 - u_1^s \\ v_2 &= u_2 - u_2^s \end{aligned} \tag{5.2.1}$$

where

y_i and v_i are deviations from the steady state into the cost function equation (5.1.2) and equations (5.1.1), (5.1.3) and (5.1.4) one obtains:

$$J = \int_{t_o}^{t_f} [\alpha(y_1^2+y_4^2) + \alpha_2 y_2^2 + \alpha_3 y_3^2 + \gamma_1 y_5^2 + \gamma_2 y_6^2 + \gamma_3 y_7^2 + \beta_1 v_1^2 + \beta_2 v_2^2 + \sum_{i=1}^{4} (\nu_i a_i^2 + \sigma_i b_i^2)]dt \tag{5.2.2}$$

Note that we set $x_2^s = 0$ because the torque angle does not change in the steady state. Also quadratic penalties are imposed on the a_i's and b_i's.

$$\dot{y}_1 = x_4^s y_2 + y_2 y_4$$

$$\dot{y}_2 = \frac{1}{M} [y_5 - S_4 x_4^s y_1 - S_5 x_3^s y_1 - K_d y_2 - S_5 x_1^s y_3 - S_4 x_1^s y_4 - S_4 y_1 y_4 - S_5 y_1 y_3 + K[u(t-t_o) - u(t-\tau)]$$

$$\begin{aligned}
\dot{y}_3 &= y_6 - A\, y_3 + C\, y_4 \\
\dot{y}_4 &= - y_1 y_2 - y_2 x_1^s \\
\dot{y}_5 &= \frac{1}{T_b} [y_7 - y_5] \\
\dot{y}_6 &= \frac{1}{T_e} [C_r\, v_2 - y_6] \\
\dot{y}_7 &= \frac{1}{T_g} [G_2 G_3\, v_1 - y_7]
\end{aligned} \tag{5.2.3}$$

$$v_1 = a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4$$

$$v_2 = b_1y_1 + b_2y_2 + b_3y_3 + b_4y_4$$

$$\dot{a}_1 = \dot{a}_2 = \dot{a}_3 = \dot{a}_4 = 0$$

$$\dot{b}_1 = \dot{b}_2 = \dot{b}_3 = \dot{b}_4 = 0$$

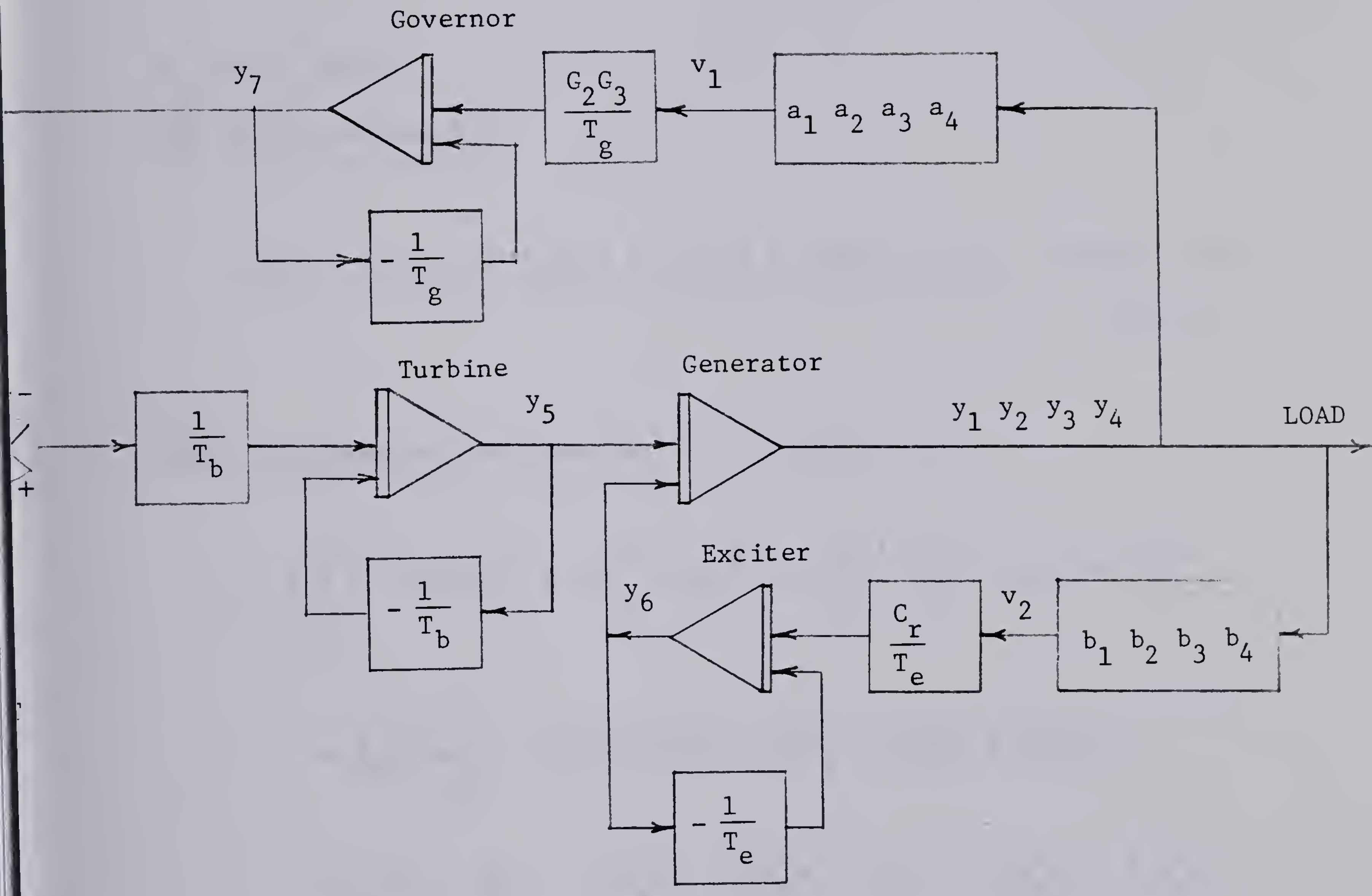

Fig. 5.2 State-Space Diagram of the Model

It should be noted that from equations (5.2.1) and (5.2.3) it follows that

$$x_5^s - S_4 x_4^s x_1^s - S_5 x_1^s x_3^s = 0$$

$$x_6^s - A x_3^s + C\, x_4^s = 0$$

at steady state.

The initial values are

$$y_1(t_o) = y_2(t_o) = y_3(t_o) = y_4(t_o) = y_5(t_o) = y_6(t_o) = y_7(t_o) = 0 \tag{5.2.4}$$

Then the augmented cost function is (Section 2.3):

$$J_1 = \int_{t_o}^{t_f} [\alpha(y_1^2+y_4^2) + \alpha_2 y_2^2 + \alpha_3 y_3^2 + \gamma_1 y_5^2 + \gamma_2 y_6^2 + \gamma_3 y_7^2 + \beta_1 v_1^2$$

$$+ \beta_2 v_2^2 + \sum_{i=1}^{4} (\nu_i a_i^2 + \sigma_i b_i^2) + \lambda_1(\dot{y}_1 - [x_4^s y_2 + y_2 y_4])$$

$$+ \lambda_2(\dot{y}_2 - \frac{1}{M}[y_5 - S_4 x_4^s y_1 - S_5 x_3^s y_1 - K_d y_2 - S_5 x_1^s y_3 - S_4 x_1^s y_4$$

$$- S_4 y_1 y_4 - S_5 y_1 y_3 + K(u(t-t_o) - u(t-\tau))]) + \lambda_3(\dot{y}_3 - y_6 + A y_3 - C y_4)$$

$$+ \lambda_4(\dot{y}_4 + y_1 y_2 + x_1^s y_2) + k_1(v_1 - (a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4))$$

$$+ k_2(v_2 - (b_1 y_1 + b_2 y_2 + b_3 y_3 + b_4 y_4)) + \lambda_5(\dot{a}_1) + \lambda_6(\dot{a}_2)$$

$$+ \lambda_7(\dot{a}_3) + \lambda_8(\dot{a}_4) + \lambda_9(\dot{b}_1) + \lambda_{10}(\dot{b}_2) + \lambda_{11}(\dot{b}_3)$$

$$+ \lambda_{12}(\dot{b}_4) + \lambda_{13}\,[\dot{y}_5 - \frac{1}{T_b}\,(y_7-y_5)] + \lambda_{14}[\dot{y}_6 - \frac{1}{T_e}\,(C_r v_2-y_6)]$$

$$+ \lambda_{15}[\dot{y}_7 - \frac{1}{T_g}\,(G_2 G_3\, v_1-y_7)]]dt$$

Integrating by parts and collecting terms which are quadratic and linear in the y_i's and a_j's and b_k's and dropping terms which do not depend on these, the augmented cost function reduces to:

$$J_1 = \int_{t_o}^{t_f} [Y^T QY + V^T RV + A^T SA + B^T TB + \Lambda^T(\dot{Y}-F)]dt$$

where

$$Y^T = [y_1\ y_2\ y_3\ y_4\ y_5\ y_6\ y_7]$$
$$V^T = [v_1\ v_2]$$
$$A^T = [a_1\ a_2\ a_3\ a_4]$$
$$B^T = [b_1\ b_2\ b_3\ b_4]$$
$$\Lambda^T = [\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\ \lambda_5\ \lambda_6\ \lambda_7\ \lambda_8\ \lambda_9\ \lambda_{10}\ \lambda_{11}\ \lambda_{12}\ \lambda_{13}\ \lambda_{14}\ \lambda_{15}]$$

$$Q = \begin{bmatrix} \alpha & & & & & & 0 \\ & \alpha_2 & & & & & \\ & & \alpha_3 & & & & \\ & & & \alpha & & & \\ & & & & \gamma_1 & & \\ & & & & & \gamma_2 & \\ 0 & & & & & & \gamma_3 \end{bmatrix}$$

$$R = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}$$

$$S = \begin{bmatrix} \nu_1 & & & 0 \\ & \nu_2 & & \\ & & \nu_3 & \\ 0 & & & \nu_4 \end{bmatrix}$$

$$T = \begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \sigma_3 & \\ 0 & & & \sigma_4 \end{bmatrix}$$

The matrices Q,R,S and T are required to be positive definite (Section 2.3) for minimum J_1.

Thus, the Hamiltonian is defined as (Section 2.3):

$$H = \alpha(y_1^2+y_4^2) + \alpha_2 y_2^2 + \alpha_3 y_3^2 + \gamma_1 y_5^2 + \gamma_2 y_6^2 + \gamma_3 y_7^2$$

$$+ \beta_1 v_1^2 + \beta_2 v_2^2 + \lambda_1(-[x_4^s y_2 + y_2 y_4])$$

$$+ \lambda_2(- \frac{1}{M} [y_5 - S_4 x_4^s y_1 - S_5 x_3^s y_1 - K_d y_2 - S_5 x_1^s y_3 - S_4 x_1^s y_4$$

$$- S_4 y_1 y_4 - S_5 y_1 y_3 + K(u(t-t_o) - u(t-\tau))])$$

$$+ \lambda_3(-y_6 + Ay_3 - Cy_4) + \lambda_4(y_1 y_2 + x_1^s y_2)$$

$$+ k_1(v_1 - (a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4))$$

$$+ k_2(v_2 - (b_1 y_1 + b_2 y_2 + b_3 y_3 + b_4 y_4)) + \lambda_5(0) + \lambda_6(0)$$

$$+ \lambda_7(0) + \lambda_8(0) + \lambda_9(0) + \lambda_{10}(0) + \lambda_{11}(0) + \lambda_{12}(0)$$

$$+ \lambda_{13} (- \frac{1}{T_b} (y_7 - y_5)) + \lambda_{14} (- \frac{1}{T_e} (C_r v_2 - y_6))$$

$$+ \lambda_{15} (- \frac{1}{T_g} (G_2 G_3 v_1 - y_7)) + \sum_{i=1}^{4} (\nu_i a_i^2 + \sigma_i b_i^2)$$

The necessary and sufficient conditons for minimum J_1 by Pontryagin's Minimum Principle are [1]:

$$\dot{Y} = -\frac{\partial H}{\partial \Lambda_1}$$

$$\dot{\Lambda}_1 = \frac{\partial H}{\partial Y} \qquad (5.2.5)$$

$$\dot{\Lambda}_2 = \frac{\partial H}{\partial A}$$

$$\dot{\Lambda}_3 = \frac{\partial H}{\partial B}$$

$$\frac{\partial H}{\partial V} = 0$$

$$\frac{\partial^2 H}{\partial V^2} > 0$$

where

$$Y^T = [y_1\ y_2\ y_3\ y_4\ y_5\ y_6\ y_7]$$

$$\Lambda_1^T = [\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\ \lambda_{13}\ \lambda_{14}\ \lambda_{15}]$$

$$\Lambda_2^T = [\lambda_5\ \lambda_6\ \lambda_7\ \lambda_8]$$

$$\Lambda_3^T = [\lambda_9\ \lambda_{10}\ \lambda_{11}\ \lambda_{12}]$$

and the boundary term is minimized with respect to

$$y_i(t_f),\ a_i(t_o),\ a_i(t_f)\ b_i(t_o),\ b_i(t_f)$$

which yields

$$\lambda_i(t_f) = 0 \qquad (i=1,2,3,4,13,14,15)$$

$$\lambda_j(t_o) = \lambda_j(t_f) = 0 \quad (j=5,6,7,8,9,10,11,12). \tag{5.2.6}$$

The optimizing equations (5.2.5) are arranged and given specifically as:

$$\dot{y}_1 = x_4^s y_2 + y_2 y_4$$

$$\dot{y}_2 = \frac{1}{M}[y_5 - S_4 x_4^s y_1 - S_5 x_3^s y_1 - K_d y_2 - S_5 x_1^s y_3 - S_4 x_1^s y_4 - S_4 y_1 y_4$$

$$- S_5 y_1 y_3 + K[u(t-t_o) - u(t-\tau)]]$$

$$\dot{y}_3 = y_6 - A\, y_3 + C\, y_4$$

$$\dot{y}_4 = - y_1 y_2 - x_1^s y_2$$

$$\dot{y}_5 = \frac{1}{T_b}[y_7 - y_5]$$

$$\dot{y}_6 = \frac{1}{T_e}[C_r v_2 - y_6]$$

$$\dot{y}_7 = \frac{1}{T_g}[G_2 G_3 v_1 - y_7]$$

$$\dot{\lambda}_1 = 2\alpha y_1 + \lambda_4 y_2 + \frac{\lambda_2}{M} S_5 y_3 + \frac{\lambda_2}{M} S_4 y_4 - k_1 a_1 - k_2 b_1$$

$$+ \frac{\lambda_2}{M} S_4 x_4^s + \frac{\lambda_2}{M} S_5 x_3^s$$

$$\dot{\lambda}_2 = 2\alpha_2 y_2 + \lambda_4 y_1 - \lambda_1 y_4 - k_1 a_2 - k_2 b_2 + x_1^s \lambda_4 - x_4^s \lambda_1 + \frac{\lambda_2}{M} K_d$$

$$\dot{\lambda}_3 = 2\alpha_3 y_3 + \frac{\lambda_2}{M} S_5 y_1 - k_1 a_3 - k_2 b_3 + A\,\lambda_3 + \frac{\lambda_2}{M} S_5 x_1^s$$

$$\dot{\lambda}_4 = 2\alpha y_4 - \lambda_1 y_2 + \frac{\lambda_2}{M} S_4 y_1 - k_1 a_4 - k_2 b_4 - C\,\lambda_3 + \frac{\lambda_2}{M} S_4 x_1^s$$

$$\dot{\lambda}_5 = 2\nu_1 a_1 - k_1 y_1$$

$$\dot{\lambda}_6 = 2\nu_2 a_2 - k_1 y_2$$

$$\dot{\lambda}_7 = 2\nu_3 a_3 - k_1 y_3$$

$$\dot{\lambda}_8 = 2\nu_4 a_4 - k_1 y_4 \qquad (5.2.7)$$

$$\dot{\lambda}_9 = 2\sigma_1 b_1 - k_2 y_1$$

$$\dot{\lambda}_{10} = 2\sigma_2 b_2 - k_2 y_2$$

$$\dot{\lambda}_{11} = 2\sigma_3 b_3 - k_2 y_3$$

$$\dot{\lambda}_{12} = 2\sigma_4 b_4 - k_2 y_4$$

$$\dot{\lambda}_{13} = 2\gamma_1 y_5 + \frac{\lambda_{13}}{T_b} - \frac{\lambda_2}{M}$$

$$\dot{\lambda}_{14} = 2\gamma_2 y_6 + \frac{\lambda_{14}}{T_e} - \lambda_3$$

$$\dot{\lambda}_{15} = 2\gamma_3 y_7 + \frac{\lambda_{15}}{T_g} - \frac{\lambda_{13}}{T_b}$$

$$v_1 = \frac{G_2 G_3 \lambda_{15}}{2\beta_1 T_g} - \frac{k_1}{2\beta_1}$$

$$v_2 = \frac{C_r \lambda_{14}}{2\beta_2 T_e} - \frac{k_2}{2\beta_2}$$

Equations (5.2.4), (5.2.6) and (5.2.7) constitute a boundary value problem for the system and these are to be solved for the feedback parameters of the system.

CHAPTER VI

COMPUTATIONS OF THE SYSTEM EQUATIONS

6.1 Computations for Turbo-Generator

Equations (5.2.7) and the feedback parameters are solved by the following method.

Equation (5.2.3) can be reduced to

$$\dot{Y} = F(y,v,t) \quad , \quad Y(t_o) = 0 \tag{6.1}$$

$$V = Z(a,b,y) \tag{6.2}$$

where

$$Y = (y_1, y_2, y_3, y_4, y_5, y_6, y_7)$$

$$V = (v_1, v_2)$$

$$a = (a_1, a_2, a_3, a_4)$$

$$b = (b_1, b_2, b_3, b_4)$$

The feedback parameters (a,b) of the system are two constant vectors to be determined such that the cost function equation (5.2.2)

$$J = \int_{t_o}^{t_f} [\alpha(y_1^2 + y_4^2) + \alpha_2 y_2^2 + \alpha_3 y_3^2 + \gamma_1 y_5^2 + \gamma_2 y_6^2 + \gamma_3 y_7^2$$

$$+ \beta_1 v_1^2 + \beta_2 v_2^2 + \sum_{i=1}^{4} (\nu_i a_i^2 + \sigma_i b_i^2)]dt$$

$$= \int_{t_o}^{t_f} L(y,\ v,\ a,\ b,\ t)dt \tag{6.3}$$

is minimized subject to $\dot{Y} = F(y,v,t)$.

This feedback control problem can be solved by a simple reformulation of the above problem. The reformulation is carried out by substitution of equation (6.2) into equation (6.1) and equation (6.3), which gives

$$J(a,b) = \int_{t_o}^{t_f} L[y,Z(a,b,y), a,b,t]dt$$

The fact that Y are the known functions results in

$$J(a,b) = \int_{t_o}^{t_f} L(y,a,b,t)dt$$

$$\dot{Y} = F(y,a,b,t)$$

Now, this feedback control problem becomes a problem of obtaining the values of parameters (a,b), which minimize the cost functional subject to the associated differential constraints. Thus we have formulated a problem of parameter optimization

$$J(a,b) = \int_{t_o}^{t_f} [\alpha(y_1^2 + y_4^2) + \alpha_2 y_2^2 + \alpha_3 y_3^2 + \gamma_1 y_5^2 + \gamma_2 y_6^2$$

$$+ \gamma_3 y_7^2 + \beta_1(a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4)^2$$

$$+ \beta_2(b_1 y_1 + b_2 y_2 + b_3 y_3 + b_4 y_4)^2$$

$$+ \sum_{i=1}^{4} (\nu_i a_i^2 + \sigma_i b_i^2)]dt$$

subject to $\dot{Y} = F(y,a,b,t)$. The feedback parameters should satisfy the two point nonlinear boundary value problem according to equation (5.2.7).

To solve this nonlinear two point boundary value problem, one has to resort to iterative numerical methods (Section 2.4) for obtaining the optimal feedback parameters and trajectories on the digital computer.

The first variation (Section 2.2) of the functional J_1 is expressed as:

$$\delta J_1 = \int_{t_o}^{t_f} [(L_a^i + \lambda^i F_a^i)\ \delta \underline{a}^i + (L_b^i + \lambda^i F_b^i)\ \delta b^i]dt$$

$$= \int_{t_o}^{t_f} [(H_a^i)^T\ \delta a^i + (H_b^i)^T\ \delta b^i]dt$$

The gradient algorithm for the cost functional J_1 with respect to the feedback parameters are given by [3]

$$a^{i+1} = a^i - \alpha\ g^i$$

$$b^{i+1} = b^i - \alpha\ h^i$$

where

$$g^i = \frac{\partial H^i}{\partial a^i} = H_a^i$$

$$h^i = \frac{\partial H^i}{\partial b^i} = H_b^i$$

Hence, if the change in a^i, b^i is selected as

$$\delta a^i = a^{i+1} - a^i = -\alpha \frac{\partial H^i}{\partial a^i}$$

$$\delta b^i = b^{i+1} - b^i = -\alpha \frac{\partial H^i}{\partial b^i}$$

with $\alpha > 0$, then

$$\delta J_1 = -\alpha \int_{t_o}^{t_f} [(H_a^i)^T H_a^i + (H_b^i)^T H_b^i] dt$$

If these nominal feedback parameters satisfy $\frac{\partial H}{\partial a}(y^i, a^i, b^i, \lambda^i, t) \cong 0$, $\frac{\partial H}{\partial b} \cong 0$ then a^i, b^i are the desired optimal feedback parameters a^*, b^*.

For $\alpha > 0$, since the increment in J_1 is governed essentially by the first variation δJ_1, this implies

$$J_1[a^i, b^i] > J_1[a^{i+1}, b^{i+1}] \tag{6.4}$$

Hence from the equation (6.4), if the first variation $||\delta J_1^i|| < \varepsilon$ is satisfied the feedback parameters will eventually converge to the optimal feedback parameters.

The gradient algorithm for the optimum are then given by

$$\begin{aligned} a_j^{i+1} &= a_j^i - \alpha g_j^i \quad (j=1,2,3,4) \\ b_j^{i+1} &= b_j^i - \alpha h_j^i \quad (j=1,2,3,4) \end{aligned} \tag{6.5}$$

Also, the gradient equations are obtained as

$$g_j = 2\nu_j a_j - k_1 y_j \quad (j=1,2,3,4)$$

$$h_j = 2\sigma_j b_j - k_2 y_j \quad (j=1,2,3,4)$$

where

$$k_1 = \frac{G_2 G_3}{T_g} \lambda_{15} - 2\beta_1 (a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4)$$

$$k_2 = \frac{C_r}{T_e} \lambda_{14} - 2\beta_2 (b_1 y_1 + b_2 y_2 + b_3 y_3 + b_4 y_4)$$

The steps to determine the optimal feedback parameters a^*, b^* on the digital computer using the steepest descent method are:

Step 1:

Consider the parameters a^i, b^i as being a piecewise-constant during $t_o \leq t \leq t_f$. Assume the initial values $a^i = a_o$, $b^i = b_o$.

Step 2:

Using a^i, b^i, integrate the state equations (5.2.3) forwards from t_o to t_f with the initial conditions $y^i(t_o) = y_o$ and store the resulting state trajectory $y^i(t)$.

Step 3:

Using the stored value of $y^i(t)$, integrate the costate equations (5.2.7) backwards from t_f to t_o with the final condition $\lambda^i(t_f) = \lambda_f$ and compute $g^i = H_a^i$, $h^i = H_b^i$ and store these functions.

Step 4:

Compute the first variation

$$||\delta J_1^i|| = ||J_1[a^i, b^i] - J_1[a^{i+1}, b^{i+1}]||.$$

Step 5:

If the stopping criterion $||\delta J_1^i||<\varepsilon$ is satisfied, terminate the iterative procedure. Then these a^i, b^i are the required optimal feedback parameters. Otherwise generate the new a^{i+1}, b^{i+1} according to equations (6.5), and update a^i, b^i by a^{i+1}, b^{i+1} and return to Step 2.

6.2 Discussion of Results

This thesis is aimed at obtaining a desired performance of the turbo-generator during the period following a distrubance by means of an optimum variation in the control inputs. The objectives of optimization are to improve the stability and to damp out any oscillations.

Both these objectives can be achieved by using a suitable function of the rotor angle of the form $\alpha(\delta - \delta^s)^2$, where α is a constant penalty factor. As shown in Chapter 5, the usual quadratic cost function in the machine variable δ can effectively be replaced by a new cost function which is quadratic in the variables $\sin\delta$ and $\cos\delta$.

In order that the rotor angle converges to a final steady value without large steady state errors, the cost function is taken as a quadratic function of the deviation of the rotor angle from the final steady state value. A function of the field flux linkage deviation $\alpha_3(x_3 - x_3^s)^2$, where α_3 is a constant penalty factor, is also included to counteract the tendency of the rotor flux to decay during the disturbance period. This helps to reduce steady state errors in the terminal voltage of the machine at the end of the optimization period.

6.2.1 Choice of Penalty Factors

Penalty factors are used in the cost function in order to

specify the relative importance of the various terms which are minimized. In this thesis, penalty factors are chosen so that the major proportion of the reduction in the value of the cost function in each iteration is obtained in the term containing δ. α_1 and α_4 are chosen so as to alter the cost function, and to compensate for the change due to the transformation as used in Chapter 4. Other penalty factors are chosen from [2].

6.2.2 Performance of Turbo-Generator

The machine is disturbed from the steady state by a torque pulse applied directly to the shaft and the generator is controlled through a linear feedback of the state variables. It is assumed that the disturbance is modelled by introducing a torque pulse in the right hand side of the torque equation of magnitude K which is increased in steps of 5% of steady state value M_t^s and a duration τ (0.5 sec). Hence in these results, the values of the variables at $t = t_o$ are the steady state values.

The optimum controls and the optimum swing curves are shown in Fig. 6.1, Fig. 6.2, Fig. 6.3, Fig. 6.5, Fig. 6.6, Fig. 6.8, Fig. 6.9 and Fig. 6.14. As shown in Fig. 6.1, Fig. 6.2 and Fig. 6.3, the variation of the rotor angle is increased as the disturbance strength K is increased.

When the disturbance is at K = 0.4, the control signal to the speeder gear drops near zero value requiring the valves to close in Fig. 6.9. This is followed by a steady signal at the frequency of the rotor oscillations; the nature of the variations depending on whether the rotor is accelerating or decelerating. The exciter voltage control at K = 0.4 is shown in Fig.

6.8. It is shown that a suitable control at K = 0.4 can be found to improve the dynamic performance of a large turbo-generator, even in the presence of practical constraints on the control system.

The swing curve A in Fig. 6.3 shows instability at K = 2.1. It is of concern where a large disturbance occurs without any controls. However, the swing curve B in Fig. 6.3 shows stability with optimum feedback controls. This system is stable since it returns to a state of equilibrium when disturbed from a state of equilibrium.

Fig. 6.4 shows a phase plane plot corresponding to Fig. 6.3. The plot A is unstable and the torque angle δ passes unstable equilibrium point δ_u without the controls. However the motion B is governed by optimum feedback controls and the accelerating torque (K = 2.1). The machine swings to accelerate, reaching its maximum position δ_m and starts to decelerate, then returning towards δ_f. The oscillations will finally stop the rotor at δ_s with damping after the disturbance period (0.5 sec.). Fig. 6.7 also shows a phase plane plot at K = 0.4.

In conclusion, the optimum feedback controls are used for the improvement of turbo-generator stability after a disturbance.

6.2.3 <u>Effectiveness of Optimum Feedback Controls</u>

The parameters of the system are the same as [2] and the disturbance period is 0.5 sec. The exciter parameters used are Te = 0.2 sec. and C_r = 1.0. An exciter system which consists of an exciter or transformer with thyristors is assumed, where a fast rate of response can be obtained. The gain can be taken as unity without loss of generality since open loop control is assumed and per unit values are used throughout.

Fig. 6.1 shows the swing curves at K = 0.4 and Fig. 6.8, Fig. 6.9 show the optimum feedback controls required. Fig. 6.10 and Fig. 6.11 show the corresponding variations of the field voltage and the mechanical torque. Fig. 6.13 shows the corresponding variation of the excitation voltage.

The high speed voltage regulator and exciter have two major effects. The first is the increase in restoring synchronizing forces made possible through the forcing of excitation and internal machine fluxes. This improves transient stability. The second is the deterioration of machine damping, resulting in dynamic instability. However, it is possible to produce some effective control, through the variation of excitation as shown in Fig. 6.13, Fig. 6.10 and Fig. 6.8.

Fig. 6.9 shows the governor action at K = 0.4 and Fig. 6.11 shows the corresponding variation of mechanical torque. In this investigation, the governor and relay combined time constant is taken as 0.2 sec. and the turbine time constant as 0.49 sec. In actual practice, the governor and relay time constants could be smaller depending on the type of hydraulic equipment; and, in fact, the turbine time constant is only representative of the high pressure cylinder delay in a compound turbine. This is effective in preventing excessive speed variations during the disturbance, and some effects are possible with fast response as shown in Fig. 6.9, Fig. 6.11 and Fig. 6.12.

Fig. 6.5 and Fig. 6.6 show the trajectory of rotor angle after 0.5 sec. The swing curves B in Fig. 6.5 and Fig. 6.6 are with

optimum feedback controls. Fig. 6.15 and Fig. 6.16 show the trajectory of optimum feedback parameters. The optimum feedback parameters are time-independent. However, it is shown that they depend on the disturbance strength K.

The curve A in Fig. 6.14 shows the swing curve at K = 0.5 with optimum feedback parameters a_i, b_i found at K = 0.5 and the curve B shows the swing curve at K = 0.5 with optimum feedback parameters a_i, b_i found at K = 0.2. The cost function on the curve A is J (opt) = 0.50186658, but the cost function on the curve B is J (cost) = 0.51556063. Replacing the parameters (K = 0.5) by the parameters (K = 0.2) at K = 0.5, the value of the cost function becomes higher.

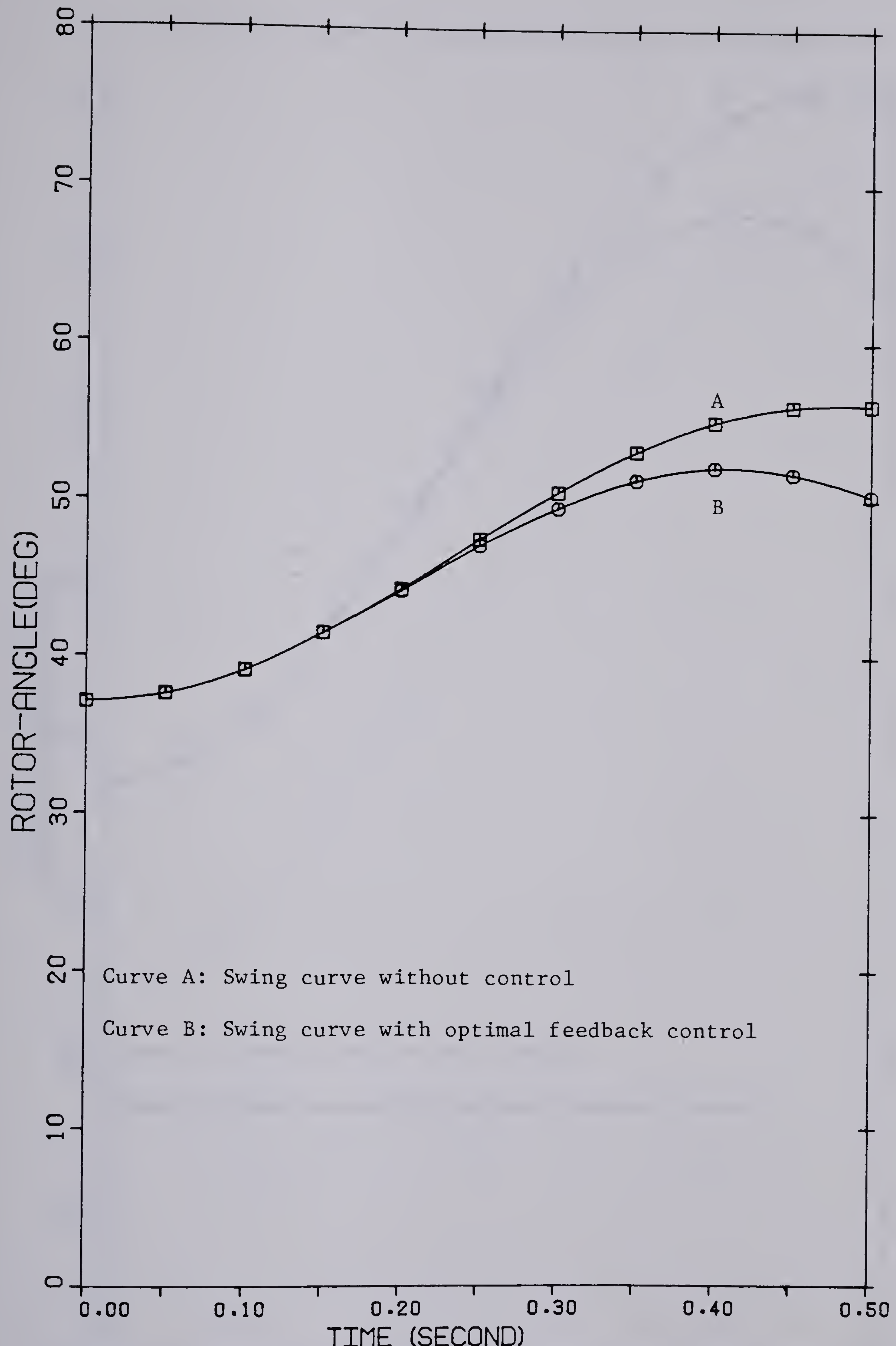

FIG. 6.1 ROTOR-ANGLE VS TIME (AT K= 0.4)

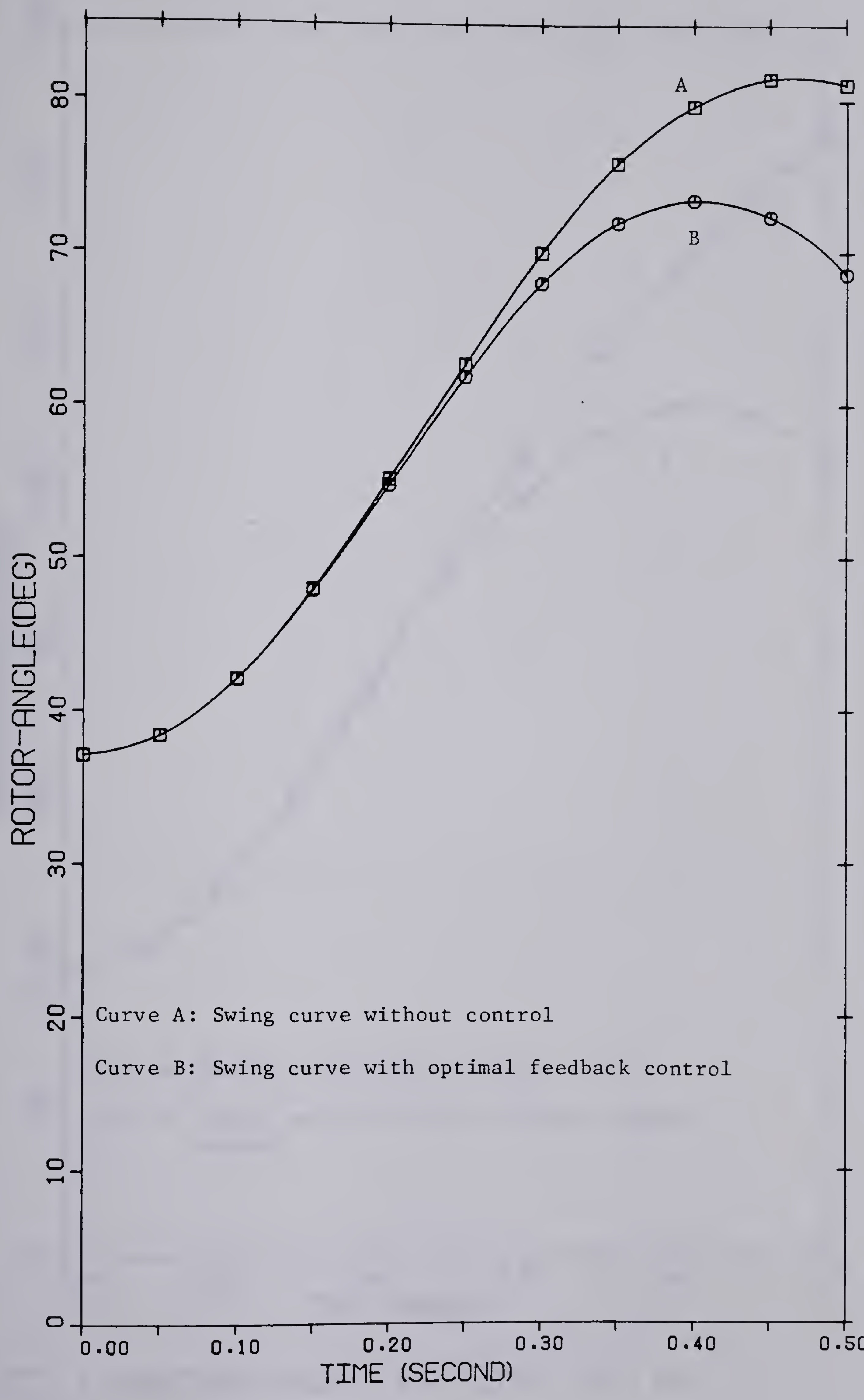

FIG. 6.2 ROTOR-ANGLE VS TIME (AT K= 1.0)

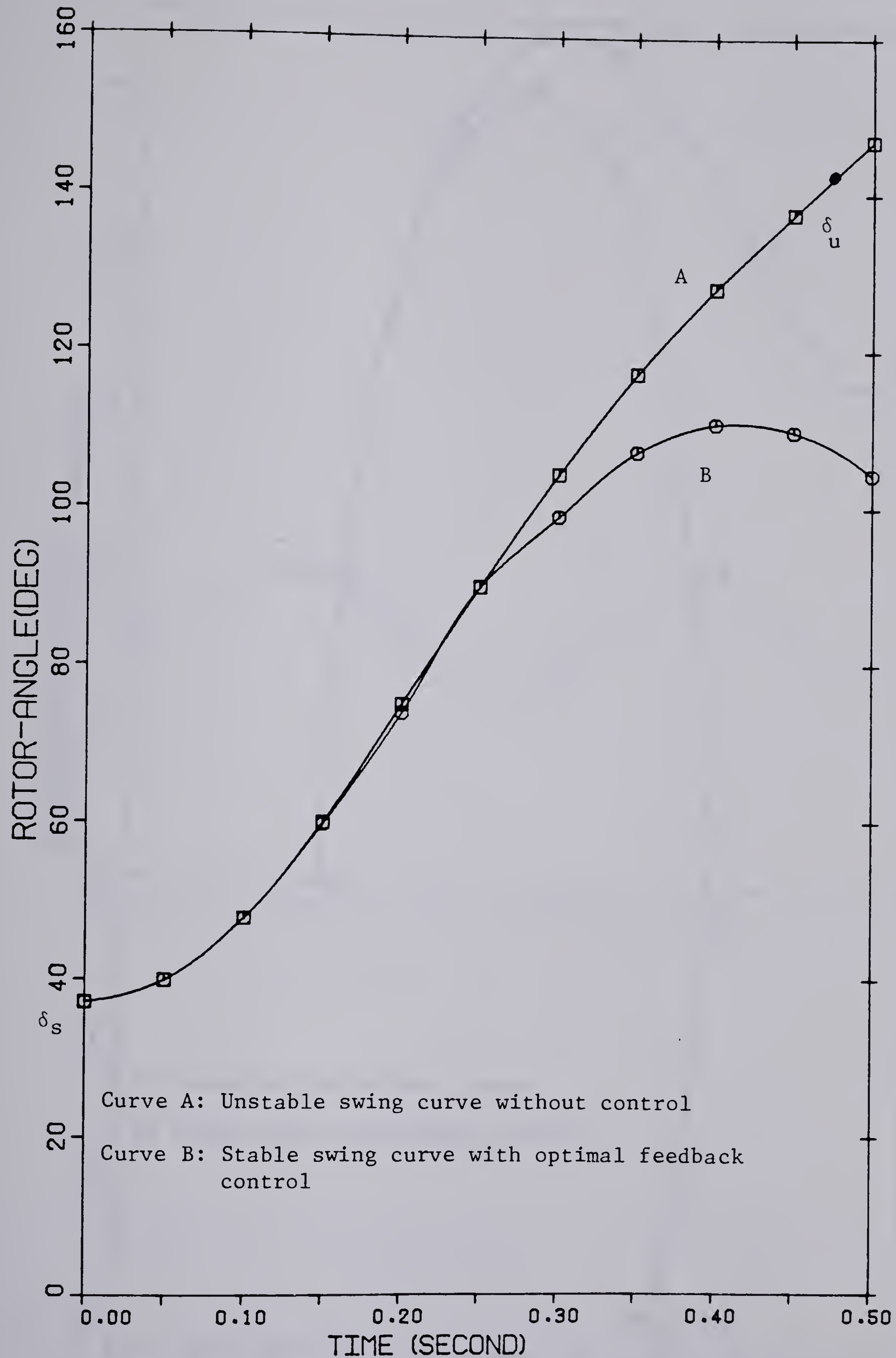

FIG. 6.3 ROTOR-ANGLE VS TIME (AT K= 2.1)

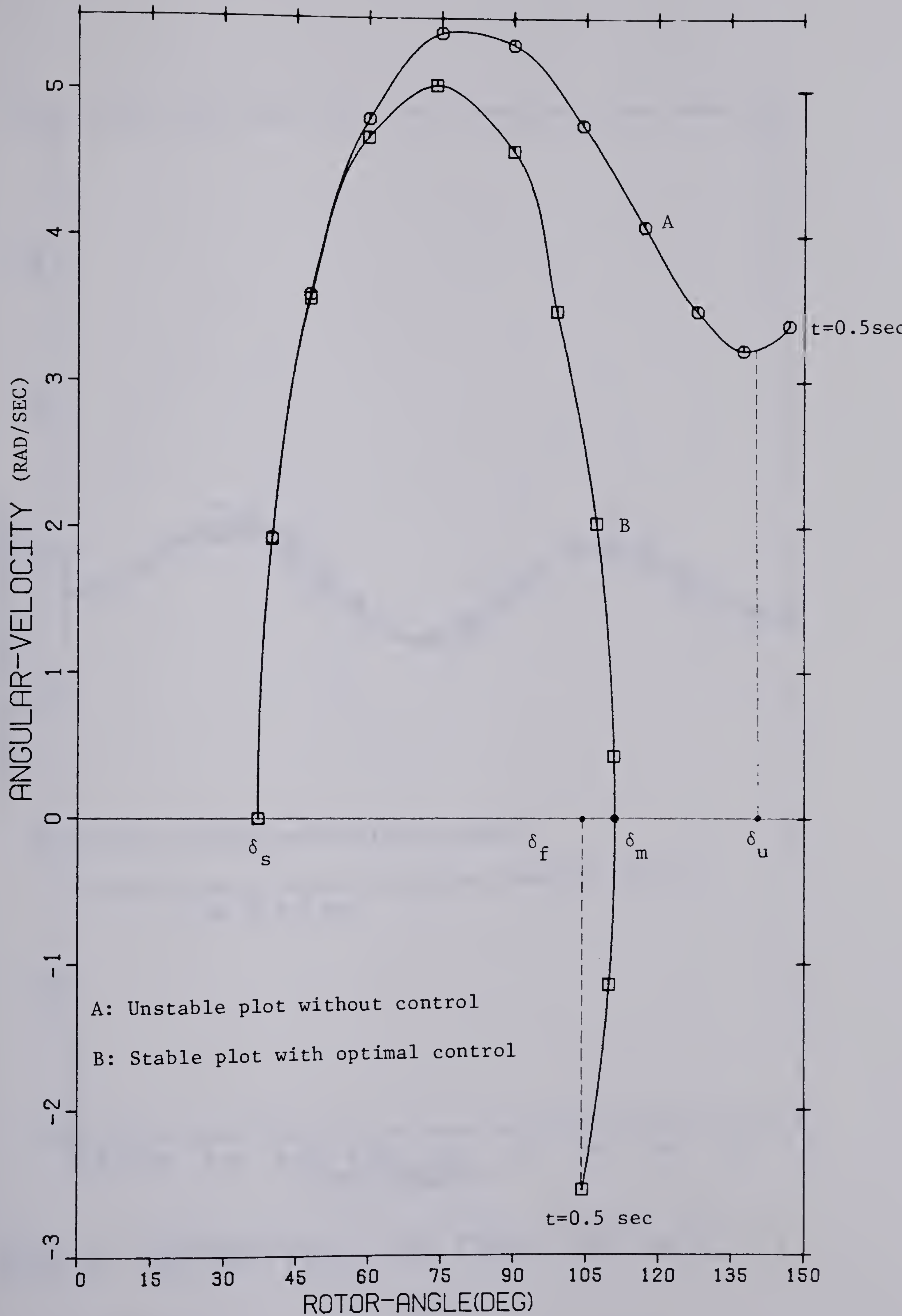

FIG. 6.4 PHASE PLANE PLOT AT K= 2.1

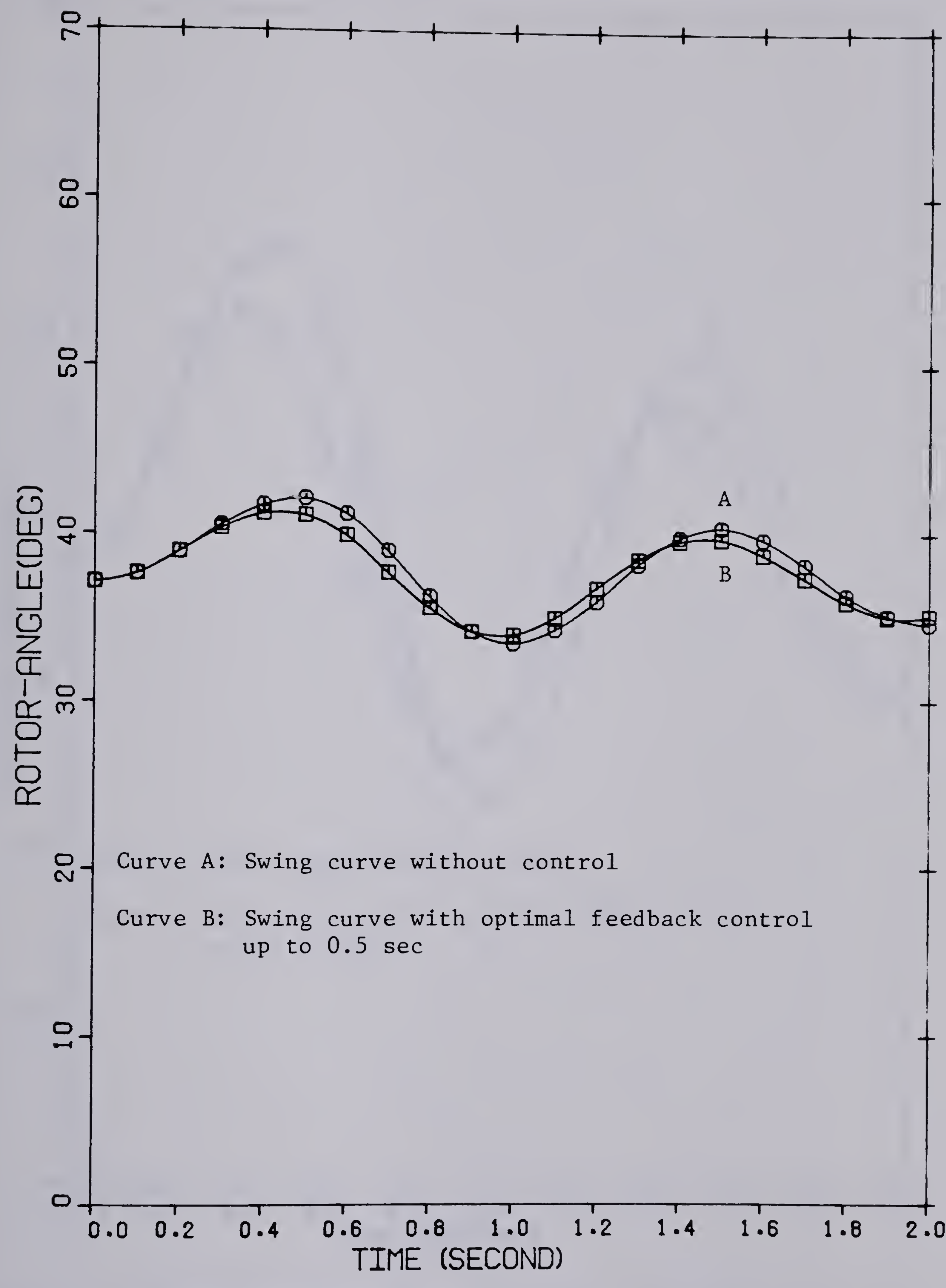

FIG. 6.5 ROTOR-ANGLE VS TIME (AT K= 0.1)

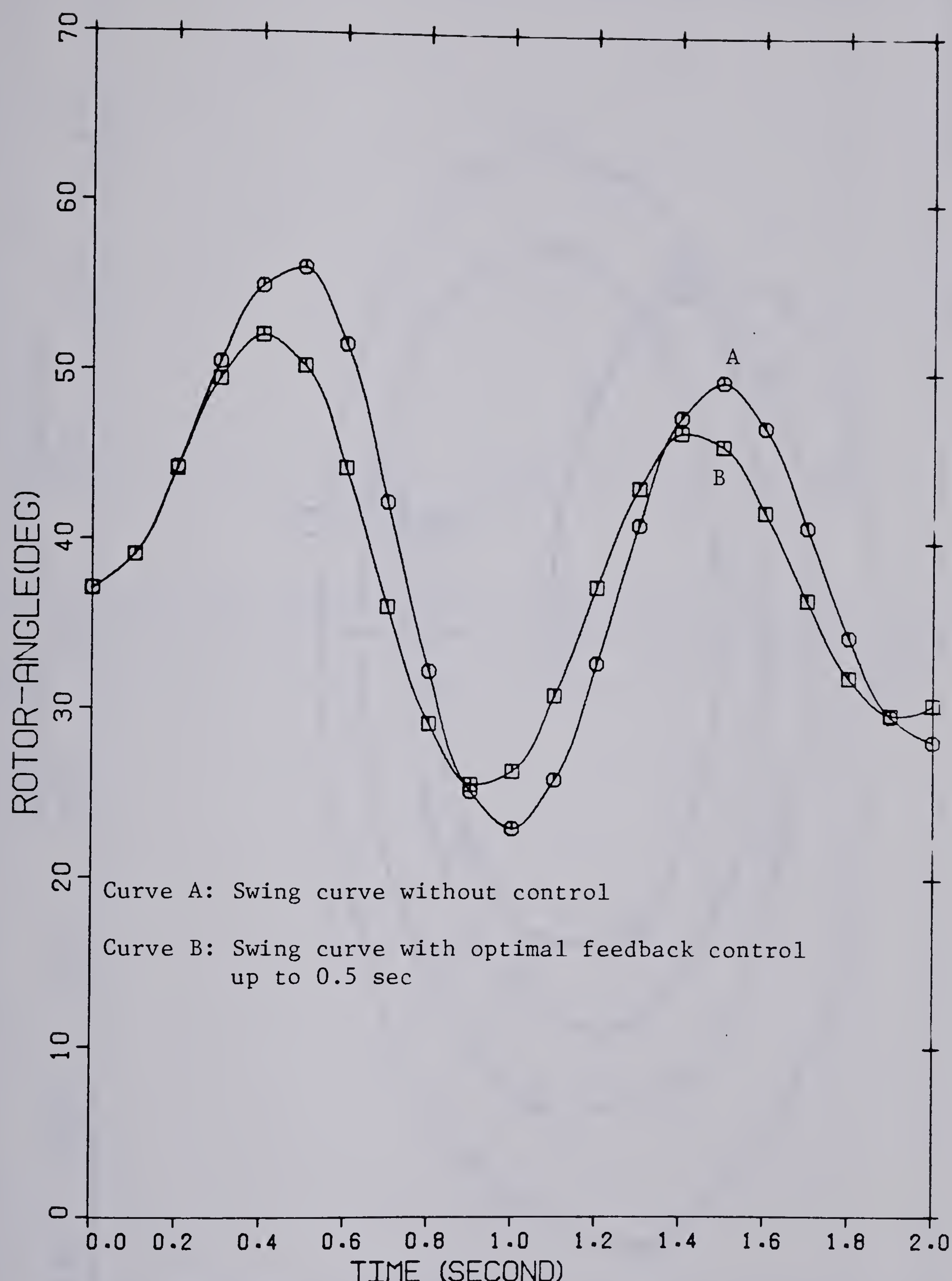

FIG. 6.6 ROTOR-ANGLE VS TIME (AT K= 0.4)

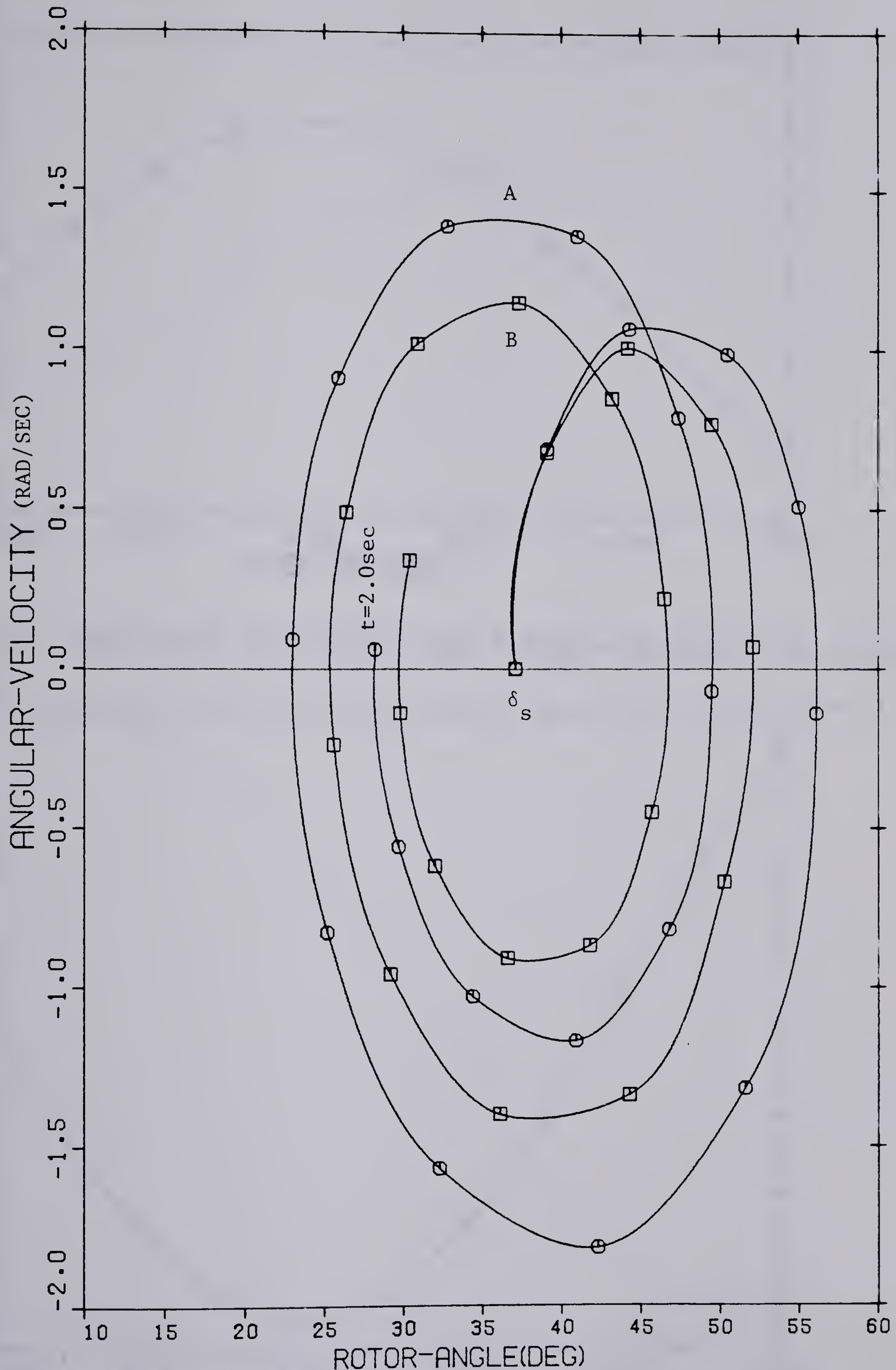

FIG. 6.7 PHASE PLANE PLOT AT K= 0.4

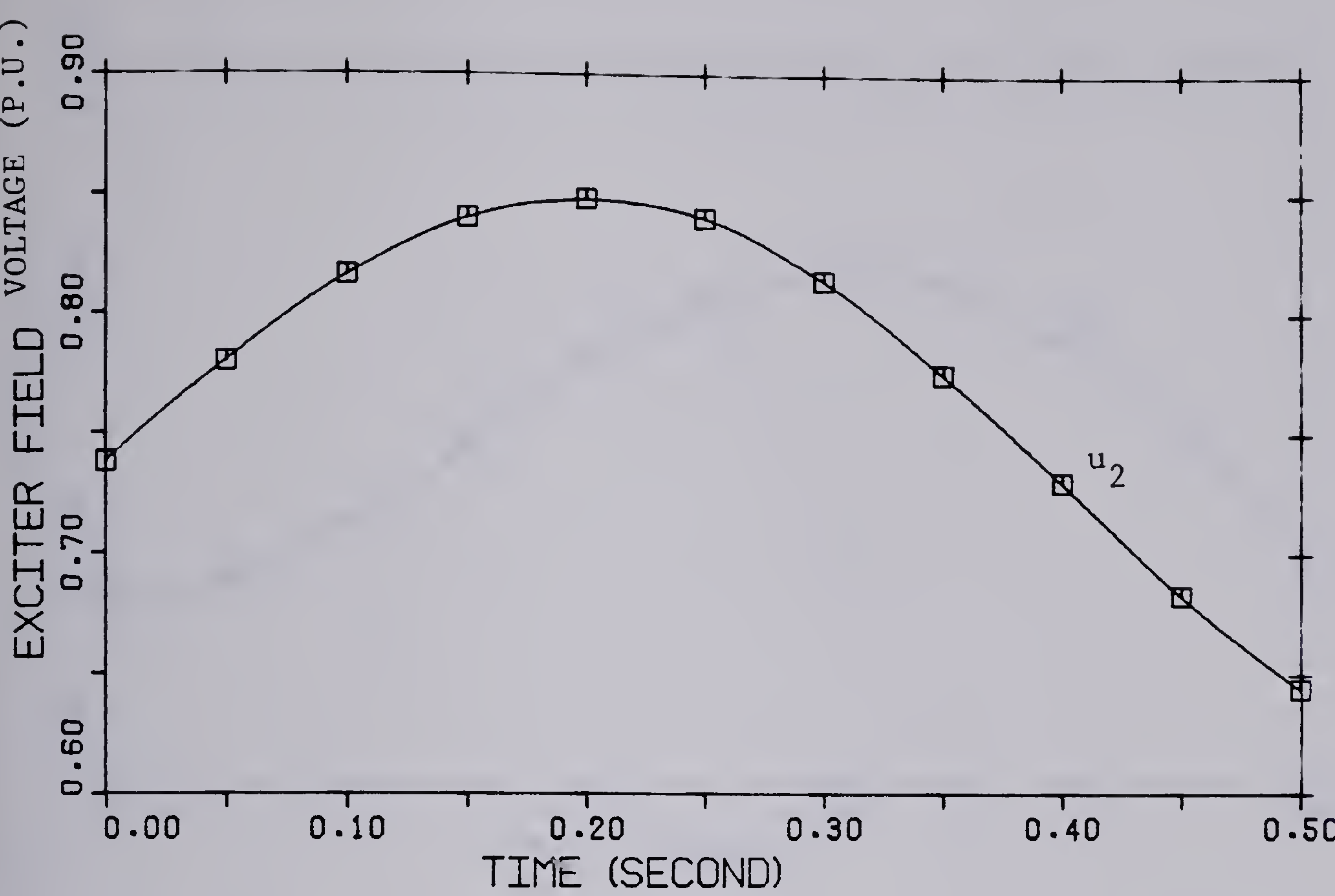

FIG. 6.8 OPTIMUM CONTROL TO TURBO-GENERATOR MODEL
(K=0.4)

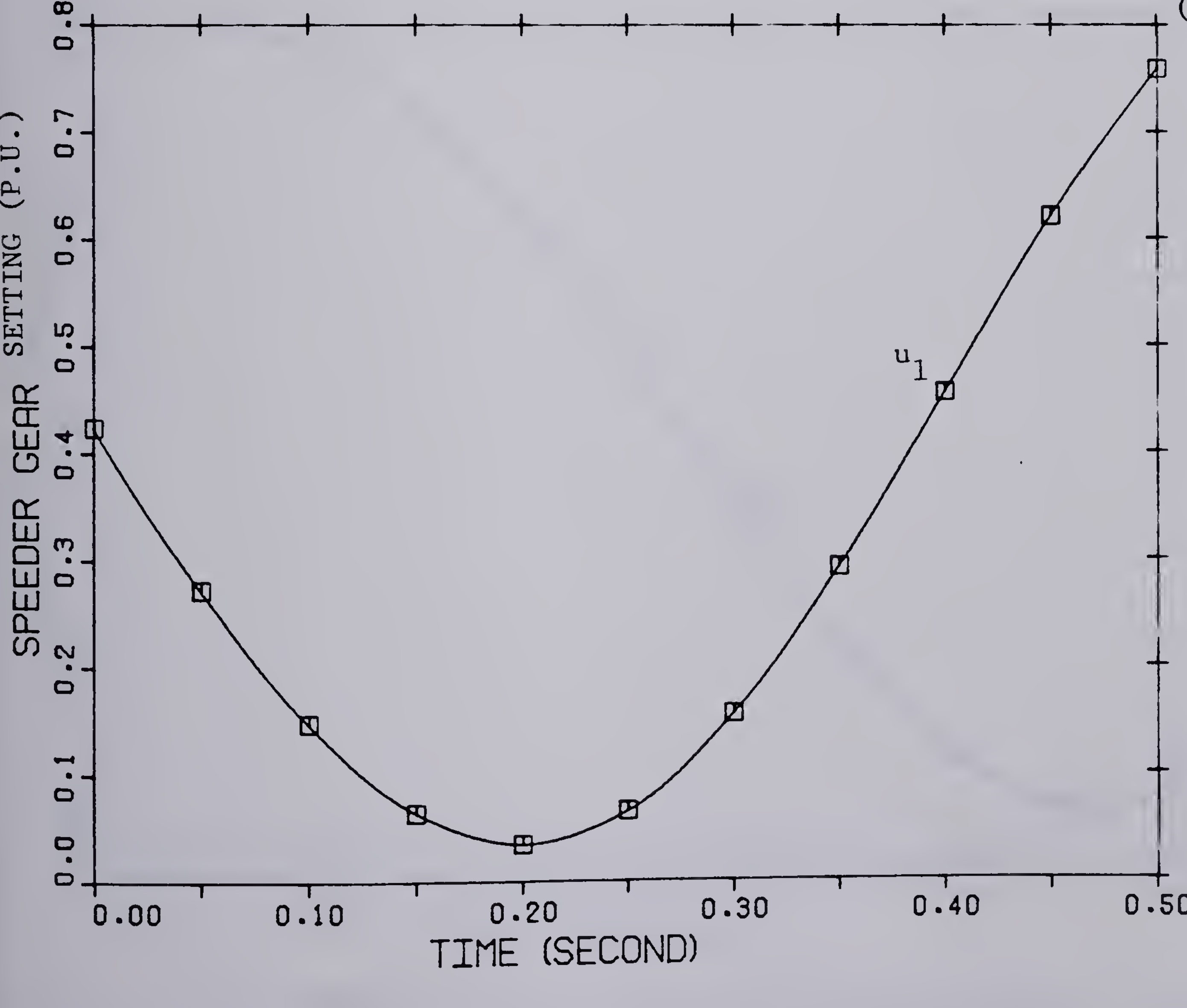

FIG. 6.9 OPTIMUM CONTROL TO TURBO-GENERATOR MODEL
(K=0.4)

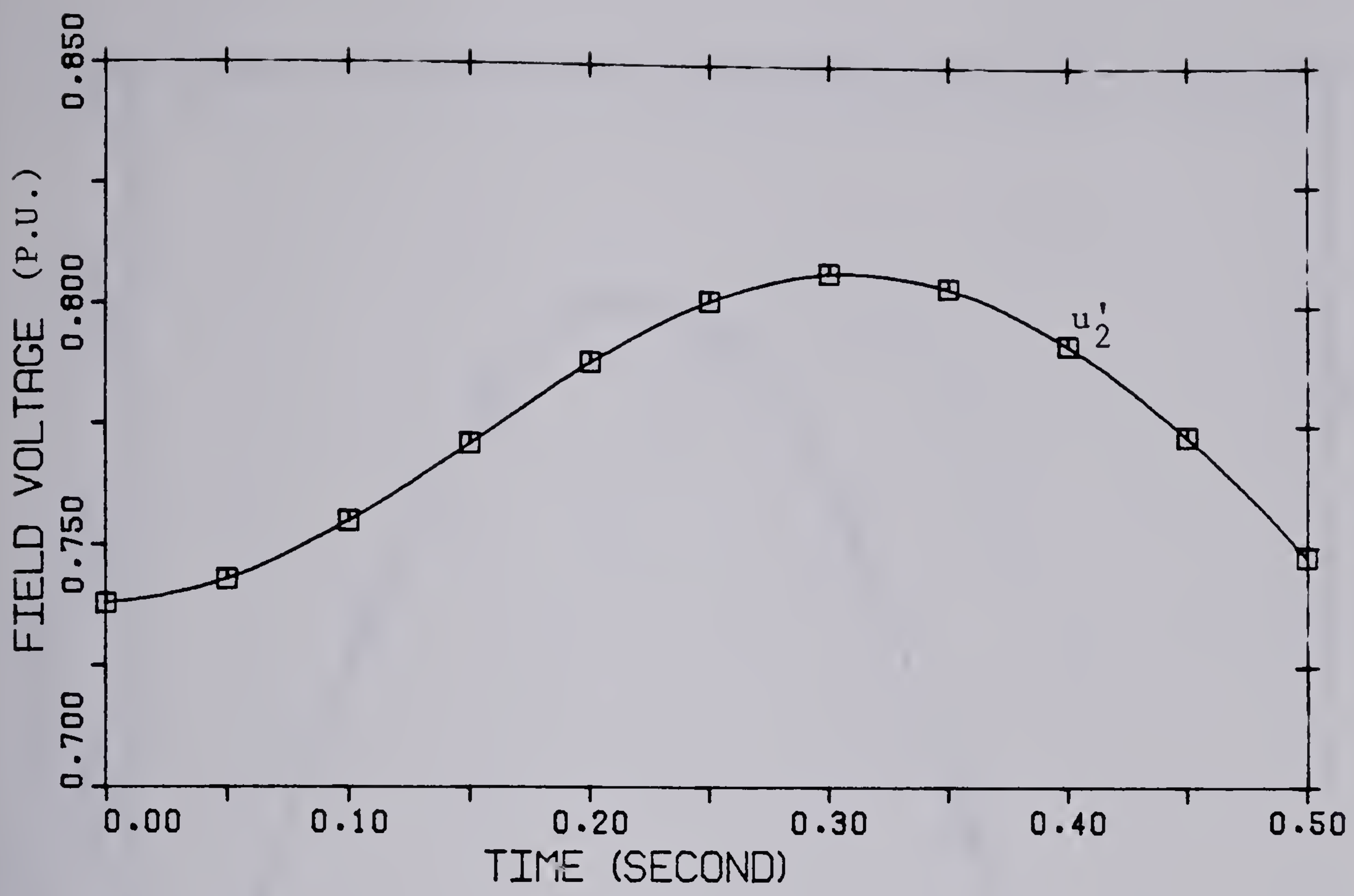

FIG. 6.10 VARIATION OF OPTIMUM FIELD VOLTAGE (K=0.4)

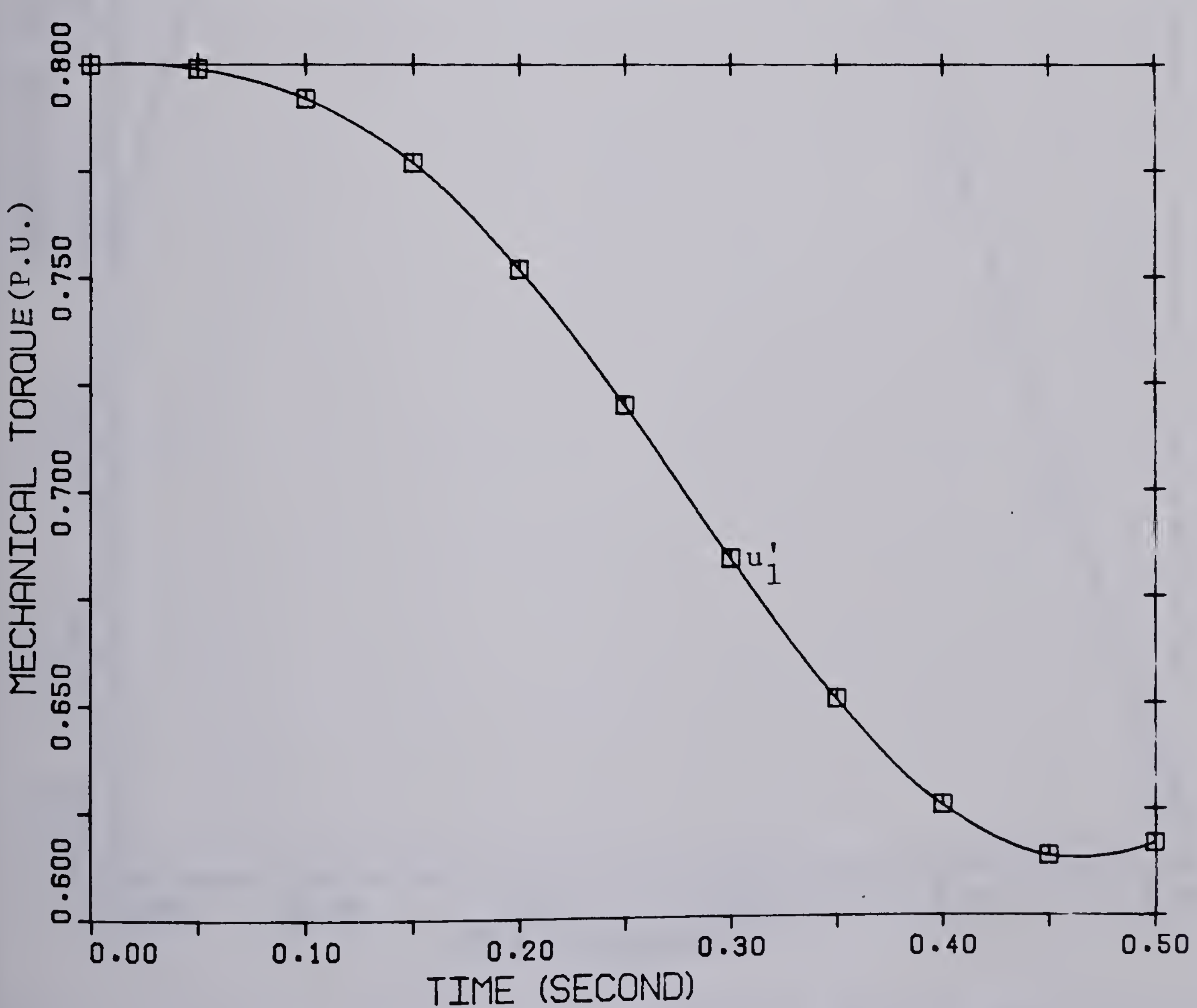

FIG. 6.11 VARIATION OF OPTIMUM MECHANICAL TORQUE (K=0.4)

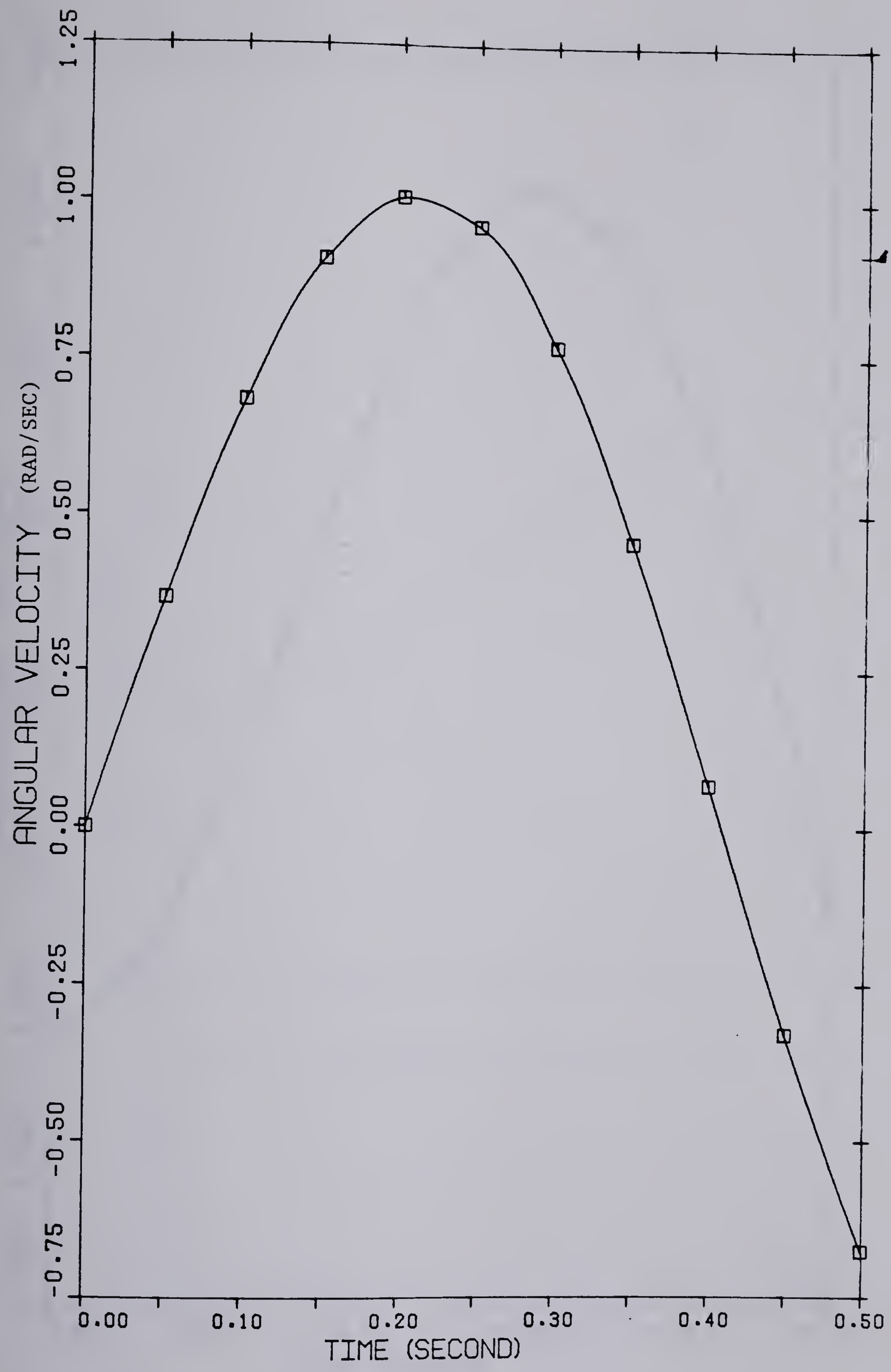

FIG. 6.12 VARIATION OF OPTIMUM ANGULAR VELOCITY (K=0.4)

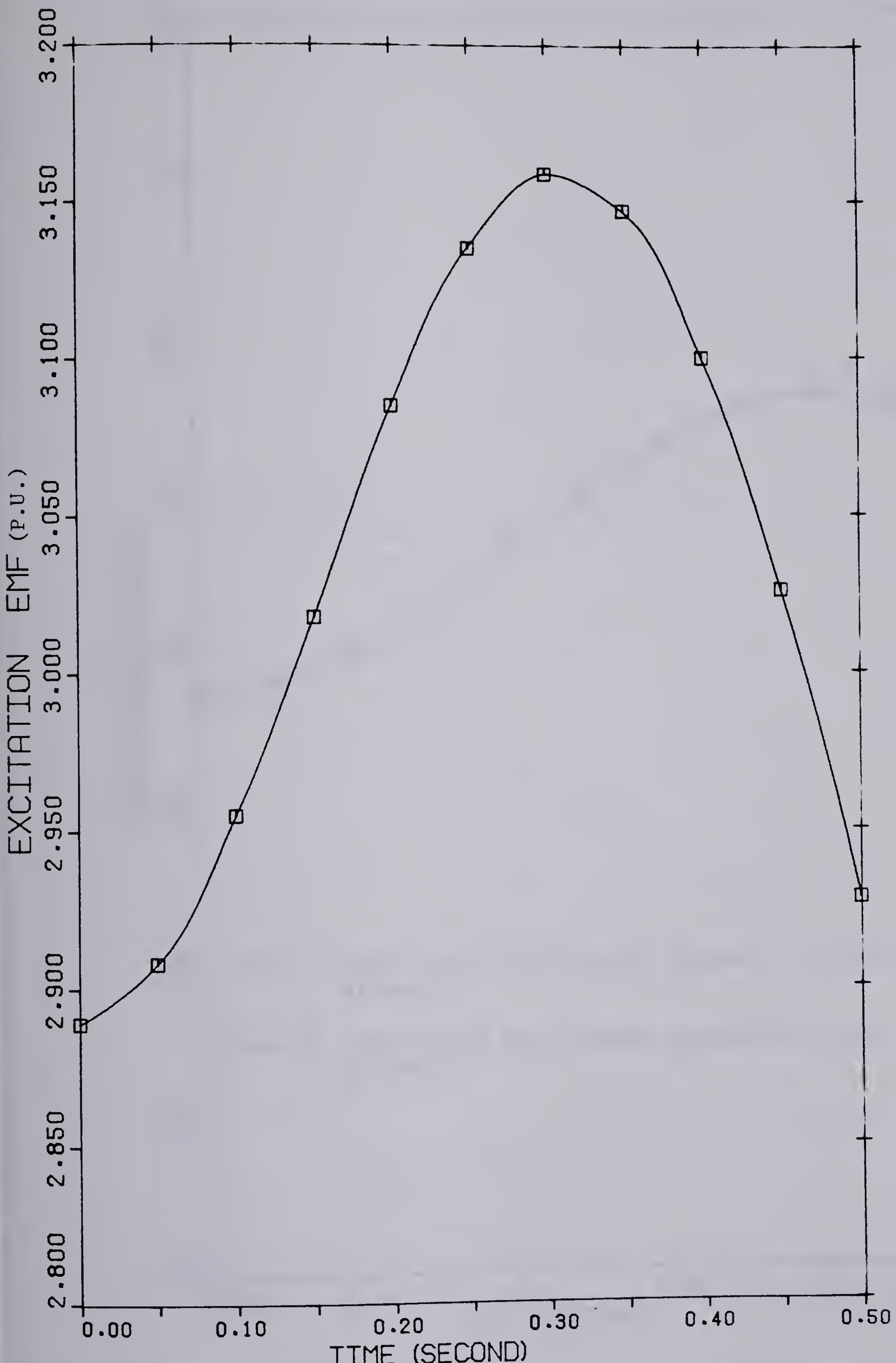

FIG. 6.13 VARIATION OF OPTIMUM EXCITATION VOLTAGE (K=0.4)

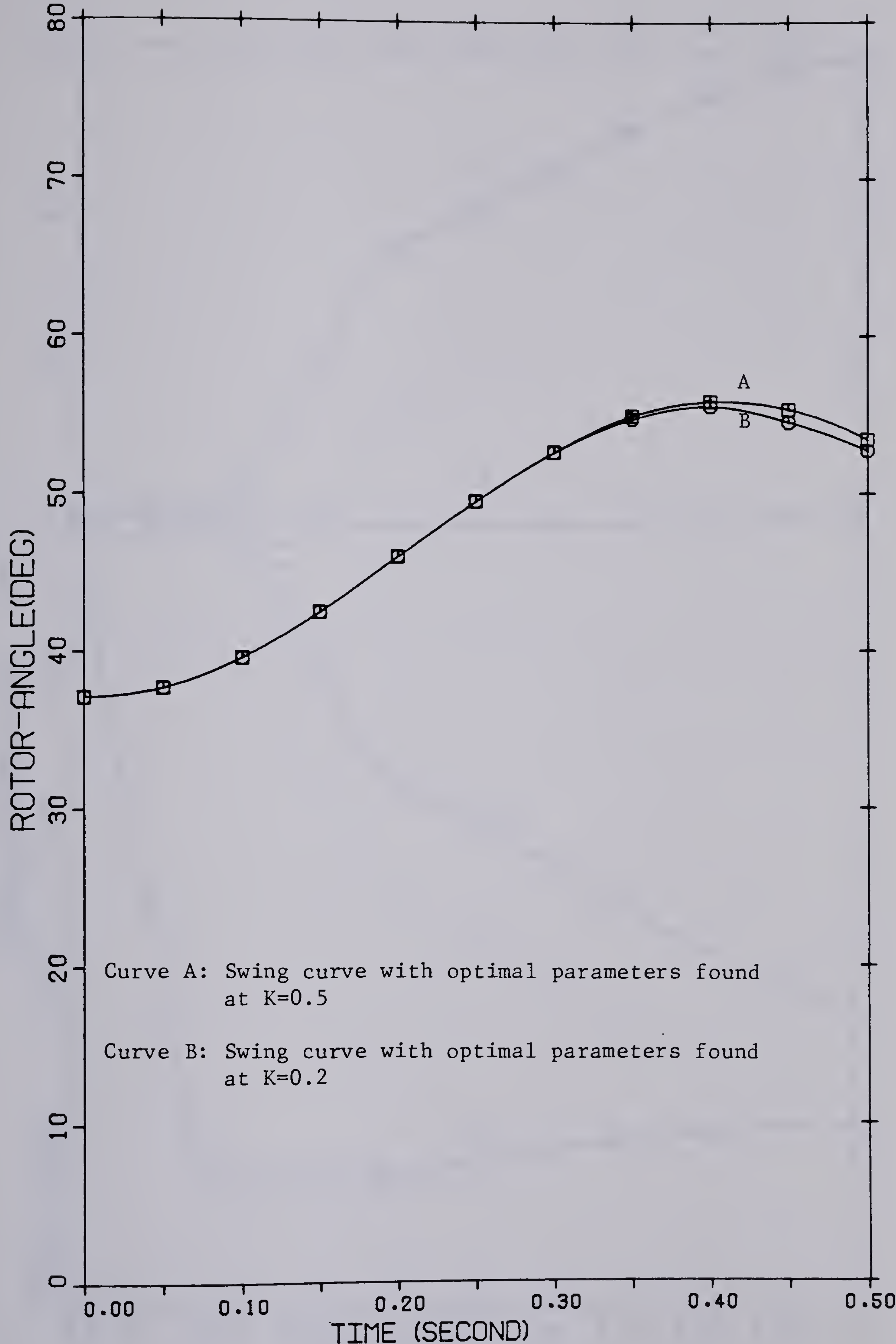

FIG. 6.14 ROTOR-ANGLE VS TIME (AT K= 0.5)

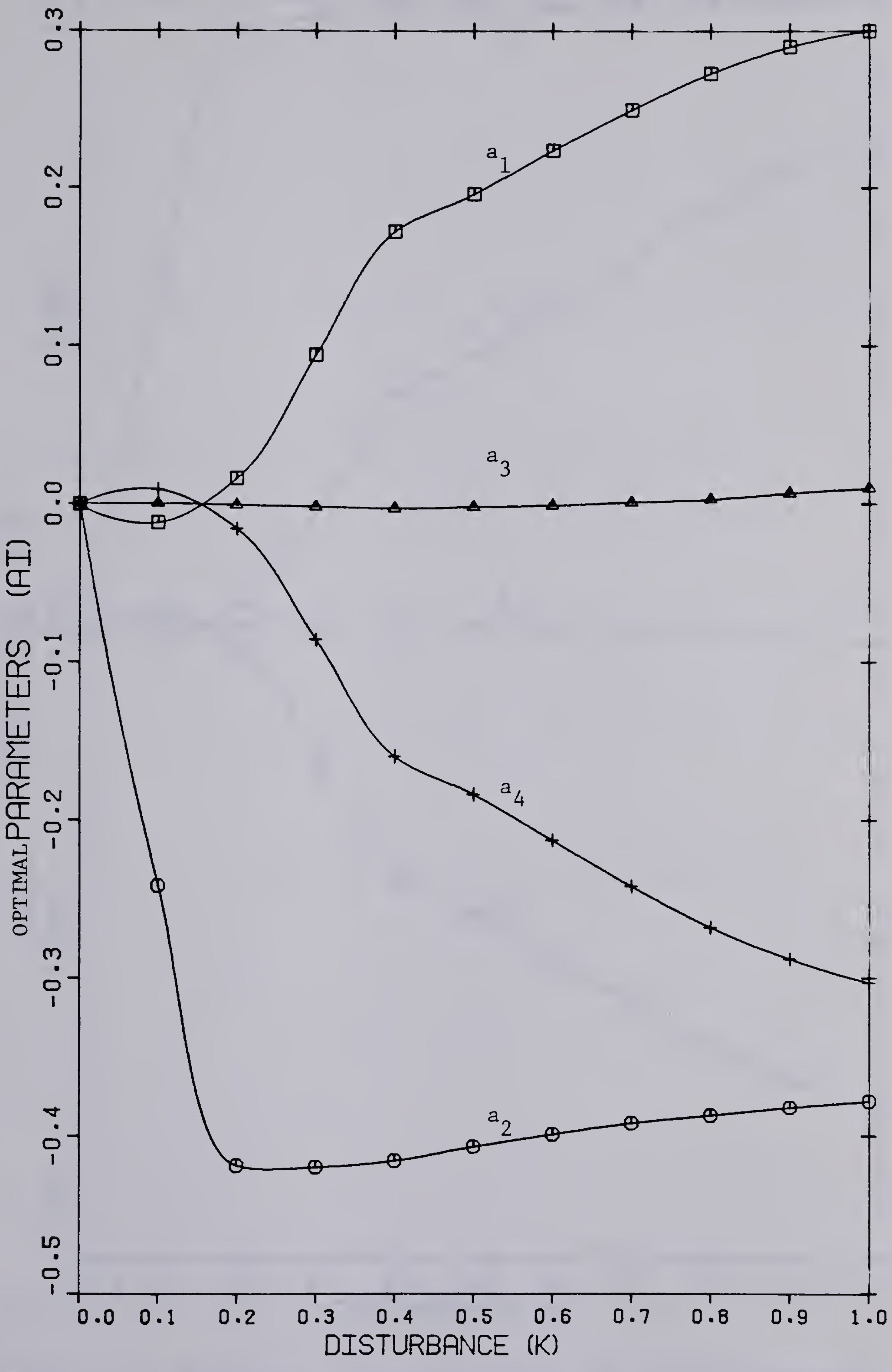

FIG. 6.15 OPTIMAL PARAMETERS VS DISTURBANCE

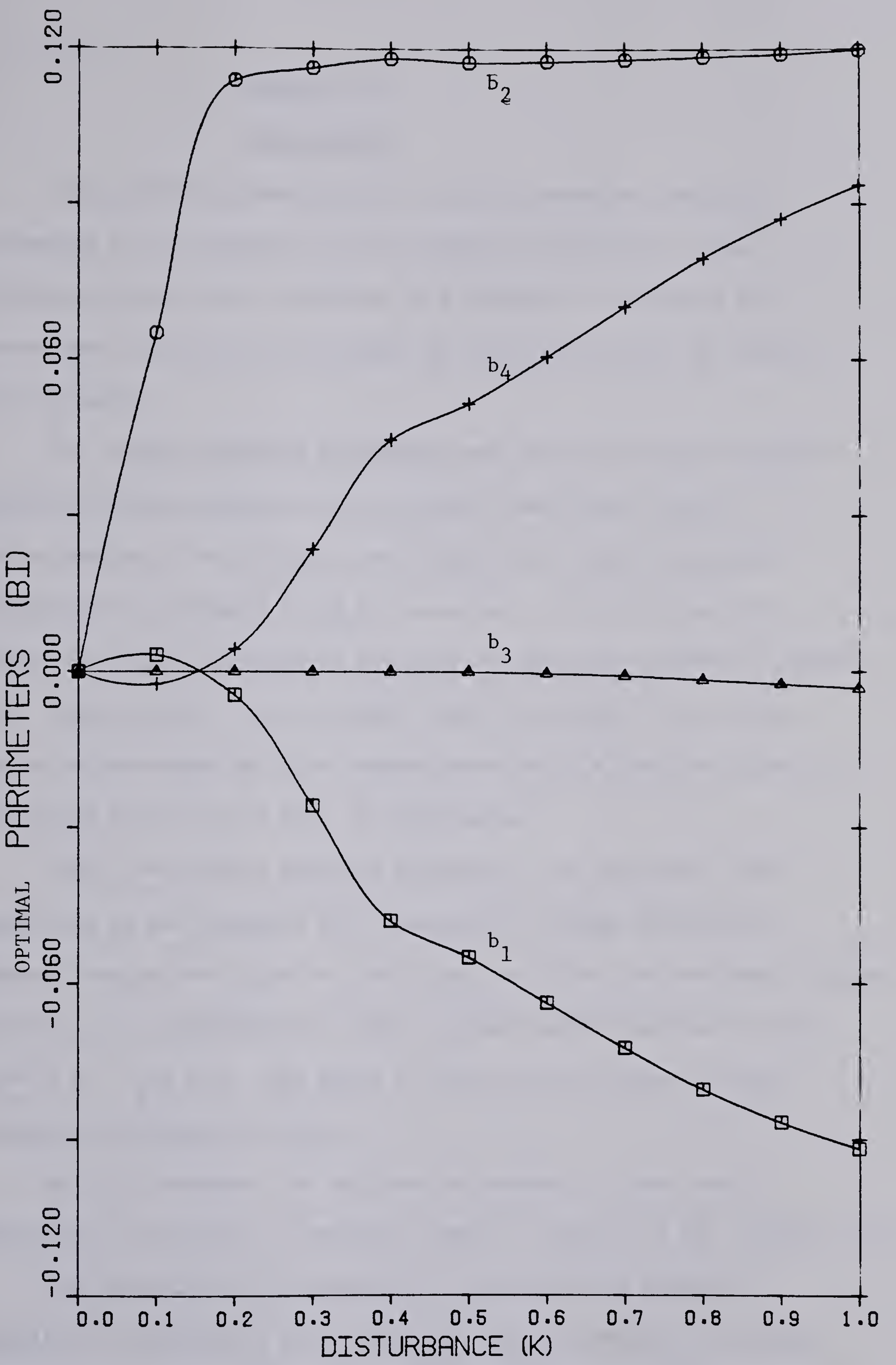

FIG. 6.16 OPTIMAL PARAMETERS VS DISTURBANCE

CHAPTER VII

CONCLUSIONS

The optimizing equations for a turbo-generator, which is connected to an infinite bus and controlled through a linear feedback of the state variables, are obtained. It should be noted that the transfer functions of the governor and the exciter are included.

The optimal feedback parameters are obtained by applying the gradient descent method to the two point nonlinear boundary value problem. The values to be obtained for these parameters depend on the strength K and the duration τ of the disturbance since the model is nonlinear contrary to the usual feedback control of a linear model. It is assumed that the strength K of torque pulse is increased by 5% of steady state value M_t and the duration τ of torque pulse is 0.5 sec. in the program.

Using the optimal feedback parameters the nonlinear state equations on the feedback are integrated by using Runge-Kutta method fowards from $t_o=0$ to $t_f=0.5$ sec. and then the nonlinear costate equations are integrated by using the same method backwards from $t_f=0.5$ to $t_o=0$ sec. The value of the cost functional is also obtained by Simpson's method.

In this research, the optimum is reached in less than 11 iterations and within a computing time of 1 minute on the AMDAHL 470/V7.

The extension of the method to a multimachine system is feasible by optimizing each machine at every time step. Although this increases computation time, the complexity of the method is not changed. The sensitivity of the system to parameter variations and the effects of damper windings are also of concern.

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A. Steady State Values of Machine Parameters Machine Constants:

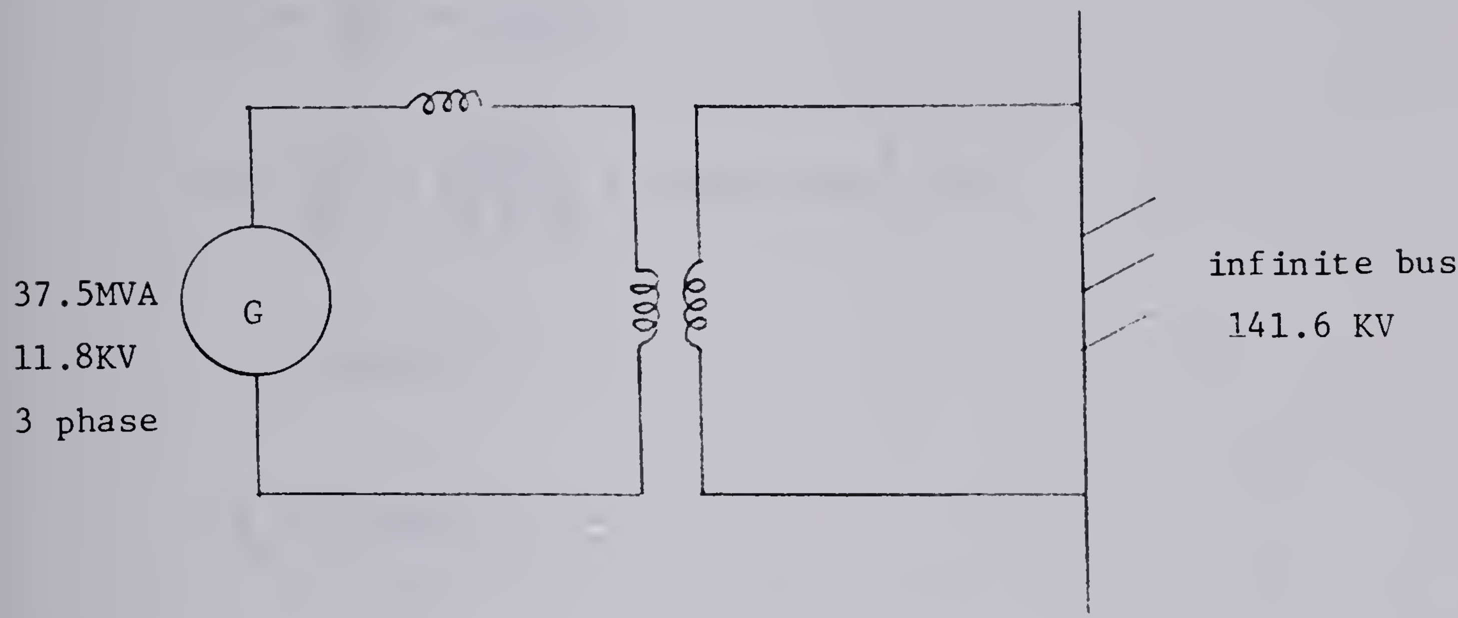
Generator rating: 37.5 MVA or 30MW at 0.8 PF.

Bus voltage (line-line) : 1 p.u.

No. of poles : 2

R.P.M. = 3000

f = 50 Hz



$x_{t_e} = 0.1265$ p.u.

on 100 MVA

$x_{t_r} = 0.354$ p.u. on 100 MVA

ratio = 11.8/141.6

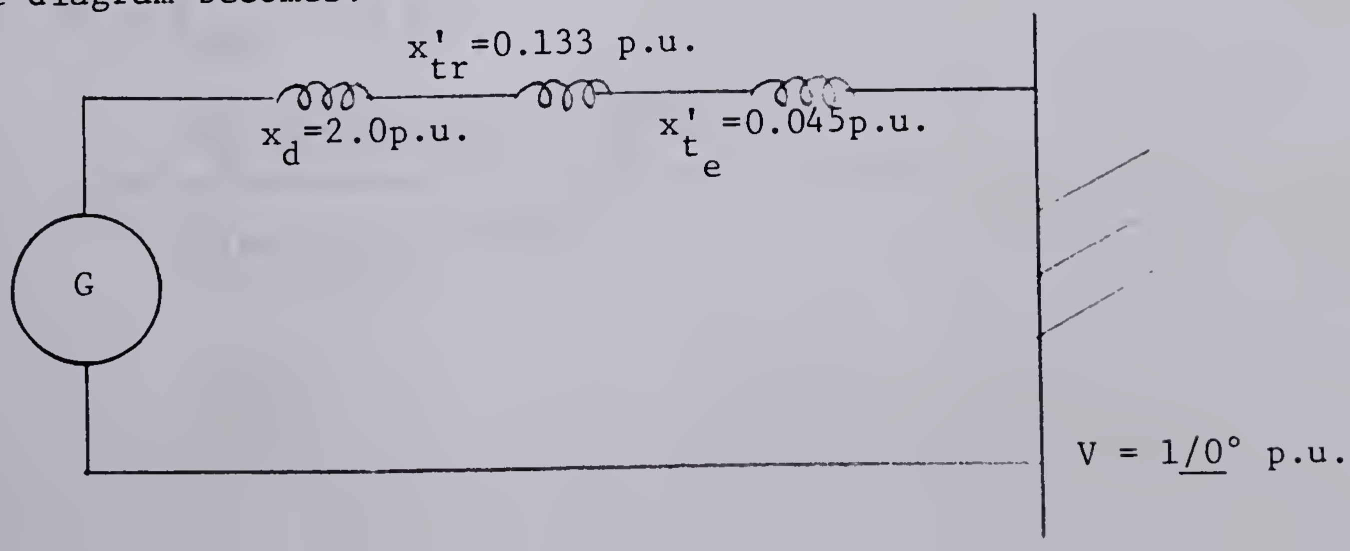
Base:

One changes from 100 MVA base to 37.5.

$$x'_{t_r} = 0.354 \times \frac{37.5}{100} \text{ p.u.} = 0.1328 \text{ p.u.}$$

$$x'_{t_e} = 0.1265 \times \frac{37.5}{100} \text{ p.u.} = 0.0475 \text{ p.u.}$$

The diagram becomes:



V_b is the line to ground phasor voltage

$$v_b = \frac{11.8}{\sqrt{3}} = 6.83 \text{ KV}$$

$$M = \frac{2H}{\omega_o} = \frac{2 \times 6.63}{2\pi \times 50} = 0.04225 \text{ sec}^2/\text{rad}$$

$$K_d = 0.02535$$

$$R_f = 0.00107 \text{ p.u.}$$

$$x_a = 0.14 \text{ p.u.}$$

$$x_{md} = 1.86 \text{ p.u.}$$

$$x_{mq} = 1.86 \text{ p.u.}$$

$$x_{fd} = 2.00 \text{ p.u.}$$

$$x_e = x'_{t_r} + x'_{t_e} = 0.133 + 0.048 = 0.181 \text{ p.u.}$$

$$V_b = E_{rms} = 1.0 \text{ p.u.}$$

$$S_4 = \frac{-V_b^2 \left(2.181 - 2.181 \times \frac{(1.86)^2}{2}\right)}{2.181\left(2.181 - \frac{(1.86)^2}{2}\right)} = -1.76$$

$$S_5 = \frac{V_b x_{md}}{\sqrt{2}\, x_{fd}} \left(\frac{1}{x_d + x_e - \frac{x_{md}^2}{x_{fd}}} \right)$$

$$= 0.707 \times \frac{1.86}{2.0} \left(\frac{1}{2.181 - \frac{1.86^2}{2}} \right) = 1.46$$

$$A = \frac{R_f\ \omega_o (x_d + x_e)}{x_{fd}(x_d + x_e - \frac{x_{md}^2}{x_{fd}})}$$

$$= \frac{0.00107 \times 50 \times 2\pi(2.181)}{2.0 \times (2.181 - \frac{1.86^2}{2})} = 0.812$$

$$C = \frac{\sqrt{2}\ V_b R_f\ \omega_o\ x_{md}}{x_{fd}(x_d + x_e - \frac{x_{md}^2}{x_{fd}})}$$

$$= \frac{\sqrt{2} \times 0.00107 \times 2\pi \times 50 \times 1.86}{2.0 \times (2.181 - \frac{1.86^2}{2})} = 0.98$$

B. <u>Steady State Value of Flux and Torque</u>

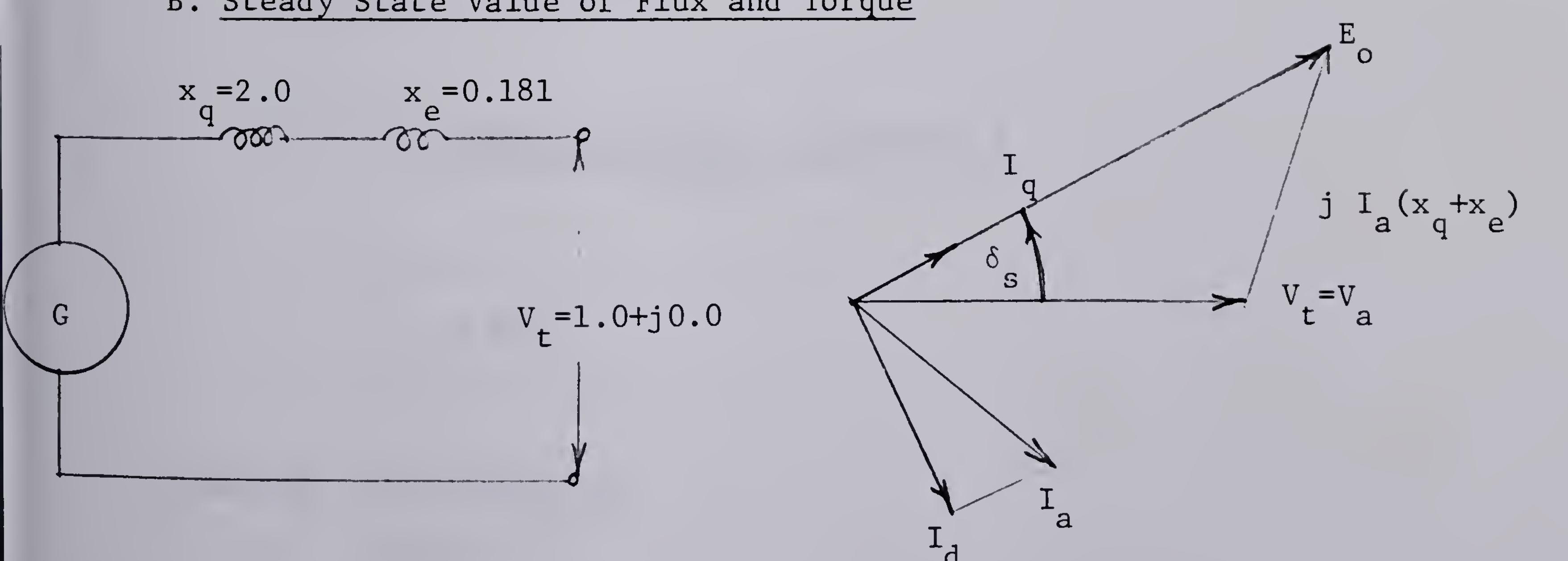


Fig. B.1 Steady State Phasor

Since PF = 0.8, $I_a = 0.8 - j0.6$

$$E_o = 1.00 + j\ 0.0 + j(2.181)\ (0.8 - j\ 0.6)$$

$$= 2.31 + j\ 1.75$$

$$= 2.89\ \underline{/37°.1}$$

Therefore $\delta_s = 37°.1$

At the steady state value of flux,

$$\frac{d^2\delta}{dt^2} = \frac{d\delta}{dt} = 0$$

$$\frac{d(\omega_o\psi_s)}{dt} = 0$$

$$\omega_o\psi_f(s) = \frac{\omega_o V_f + C\cos\delta}{A}$$

$$= \frac{100\pi(2.35\text{x}10^{-3}) + 0.98\cos 37°.1}{0.812}$$

$$= 1.874.$$

From the torque equation,

$$M_t(s) = \frac{S_4}{2} \sin 2\delta_s + S_5\, \omega_o \psi_f(s) \sin\delta_s$$

$$= \frac{-1.76}{2} \sin(2x37°.1) + 1.46 \times 1.874 \times \sin 37°.1$$

$$= 0.8$$

$$V_f(s) = \frac{E_o R_f}{x_{md}} = \frac{\sqrt{2} \times 2.89 \times 0.00107}{1.86}$$

$$= 2.35 \times 10^{-3} \text{ p.u.}$$

Let $\omega_o \psi_f = x_3$

$u_1' = M_t$

$u_2' = \omega_o V_f$,

then the equations are:

$$\frac{d^2\delta}{dt^2} = \left[\frac{u_1'}{M} - \frac{S_4}{M} \sin\delta \cos\delta - \frac{S_5}{M} x_3 \sin\delta - \frac{K_d}{M} \frac{d\delta}{dt} \right]$$

$$\frac{dx_3}{dt} = u_2' - Ax_3 + C\cos\delta$$

where

$M = 0.04225$ p.u.

$\frac{K_d}{M} = 0.02535/0.04225 = 0.6$ p.u.

$\frac{S_4}{M} = -1.76/0.04225 = -41.65$ p.u.

$$\frac{S_5}{M} = 1.46/0.04225 = 34.5 \text{ p.u.}$$

$A = 0.812$ p.u.

$C = 0.98$ p.u.

$\delta\,(s) = 37°.1$

$$\frac{d\delta\,(s)}{dt} = 0.0$$

$x_3(s) = 0.0$

$u_1'(s) = 0.8$

$u_2'(s) = 2.35 \times 10^{-3} \times 100\pi = 0.738$

$u_1(s) = Y_o(s) = 0.424$

$u_2(s) = \omega_o V_{ex}(s) = 0.738$

Also these parameters are used in all the optimal feedback control studies.

$G_2 = 1.33$

$G_3 = 1.42$

$T_g = 0.2$ sec.

$T_b = 0.49$ sec.

$C_r = 1.0$

$T_e = 0.2$ sec.

For the cost function penalty one takes:

$\alpha \;= 2.50$

$\alpha_2 = 1.00$

$\alpha_3 = 0.10$

$\beta_1 = 1.00$

$\beta_2 = 1.00$

$\gamma_1 = 0.01$

$\gamma_2 = 0.01$

$\gamma_3 = 0.01$

$\nu_1 = 0.001$

$\nu_2 = 0.001$

$\nu_3 = 0.001$

$\nu_4 = 0.001$

$\sigma_1 = 0.001$

$\sigma_2 = 0.001$

$\sigma_3 = 0.001$

$\sigma_4 = 0.001$

C. Expression of Terminal Voltage in Terms of State Variables

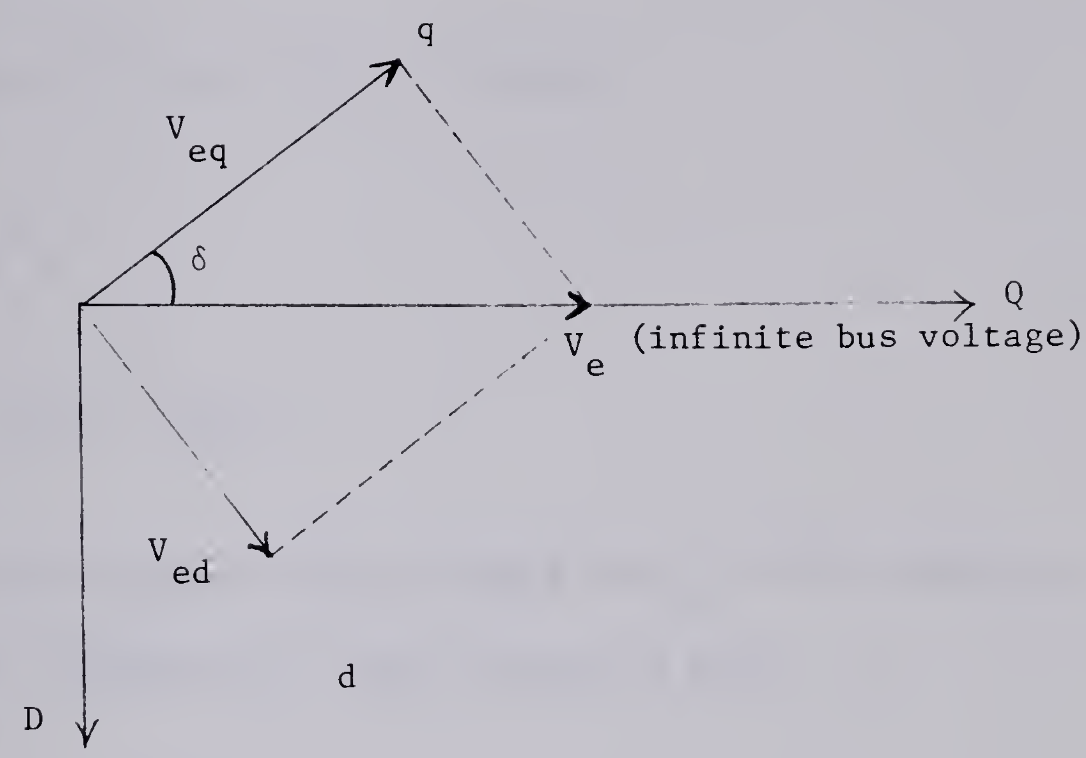


Fig. C.1 Phasor Diagram

$U_{rms} = 1.0 + j\ 0.0$ (p.u.)

$V_e = \sqrt{2}\ U_{rms}$

$V_{ed} = \sqrt{2}\ U_{rms} \sin \delta$

$V_{eq} = \sqrt{2}\ U_{rms} \cos \delta$

Under the assumptions that $\omega \approx \omega_o$, $\dot{i}_d \approx 0$. $i_q \approx 0$, $r_e \approx 0$ and $r_a \approx 0$, we have

$$V_d = V_{ed} - x_e\ i_q$$
$$V_q = V_{eq} - x_e\ i_d \qquad (C.1)$$

$$V_d = \omega_o\ \psi_q$$
$$V_q = -\ \omega_o\ \psi_d \qquad (C.2)$$

$$\omega_o \psi_d = x_d i_d + x_{ad} i_f$$
$$\omega_o \psi_q = x_q i_q \qquad (C.3)$$

Substituting (C.3) into (C.2), it becomes:

$$V_d = x_q \; i_q$$
$$V_q = - x_d \; i_d - x_{ad} \; i_f \qquad (C.4)$$

where V_d is the terminal d-axis voltage and V_q is the terminal q-axis voltage. Using $\omega_o \psi_f = x_{ad} i_d + x_f i_f$, we have

$$V_q = - x_d i_d = \frac{x_{ad}}{x_{fd}} (\omega_o \psi_f - x_{ad} i_d)$$

and hence (C.4) becomes:

$$V_d = x_q \; i_q$$

$$V_q = (\frac{x_{ad}^2}{x_{fd}} - x_d) \; i_d - \frac{x_{ad}}{x_{fd}} \omega_o \psi_f$$

Solving for i_d and i_q, we obtain

$$i_d = (V_q + \frac{x_{ad}}{x_{fd}} \omega_o \psi_f) \; / \; (\frac{x_{ad}^2}{x_{fd}} - x_d)$$
$$i_q = \frac{V_d}{x_q} \qquad (C.5)$$

Substituting (C.5) into (C.1), we obtain:

$$V_d = \frac{\sqrt{2}\, U_{rms}\, x_q}{x_q + x_e} \sin \delta$$

$$V_q = \sqrt{2}\, U_{rms} \frac{x_{ad}^2 - x_d\, x_{fd}}{x_{ad}^2 - (x_d + x_e)\, x_{fd}} \cos \delta$$

$$+ \frac{x_e\, x_{fd}}{x_{ad}^2 - (x_d + x_e)\, x_{fd}} \omega_o \psi_f$$

Data: $x_d = 2.0$, $x_q = 2.0$, $x_e = 0.181$, $x_{ad} = 1.86$, $x_{fd} = 2.0$

$\delta = 37.1°$, $\omega_o \psi_f = -1.874$

Solving for V_d and V_q by using the datas, we obtain:

$V_d = 0.782$ (p.u.)

$V_q = 1.374$ (p.u.)

Therefore

$$V_t = \frac{\sqrt{V_d^2 + V_q^2}}{\sqrt{2}} = 1.118 \text{ (p.u.)} \tag{C.6}$$

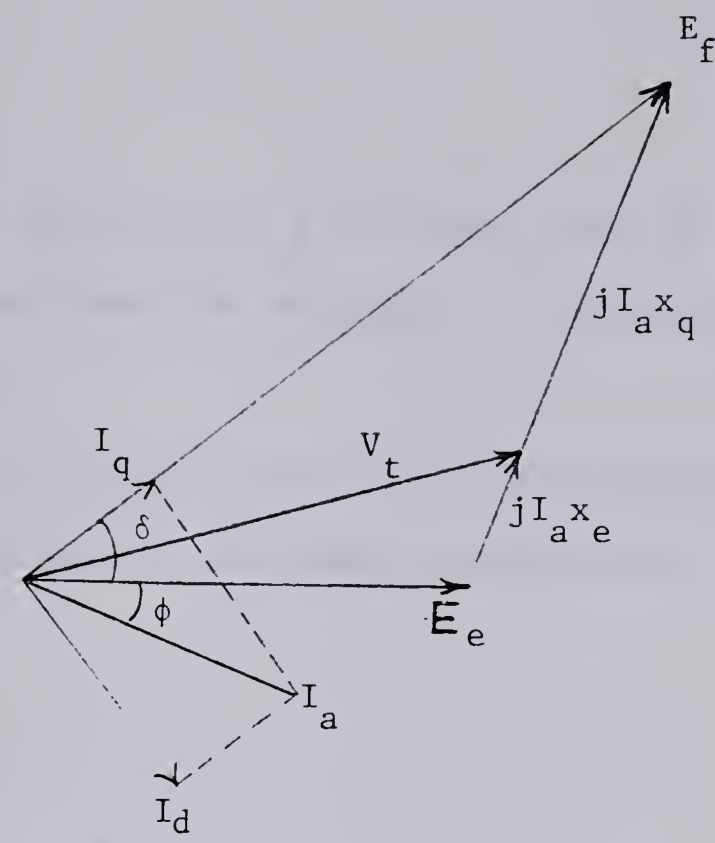


Fig. C.2 Generator Phasor Diagram

Therefore:

$$
\begin{aligned}
V_t &= E_f - j\, I_a\, x_q \\
&= E_e + j\, I_a\, x_e \\
&= (1.0 + j\,0.0) + j\,(0.8 - j\,0.6) \times 0.181 \\
&= 1.1086 + j\,0.1448 \\
&= 1.118 \underline{/7.44^\circ} \qquad \text{(C.7)}
\end{aligned}
$$

Comparing (C.6) with (C.7), the generator terminal voltages are the same. Thus, the generator terminal voltage depend on $\omega_o \psi_f$ and δ.

D. Results

It is assumed that the strength K of torque pulse is increased by 5% of steady state value M_t and the duration τ of torque pulse is 0.5 sec. in the program.

A_i, B_i, Y_i, P_i and V_i in the results are corresponding to a_i, b_i, y_i, λ_i, and v_i in the body of the thesis respectively.

K= 0.0

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0
0.30	0.0	0.0	0.0	0.0	0.0	0.0
0.35	0.0	0.0	0.0	0.0	0.0	0.0
0.40	0.0	0.0	0.0	0.0	0.0	0.0
0.45	0.0	0.0	0.0	0.0	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.0

Table 1

K= 0.05

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.34735376	0.00500644
0.05	0.00091222	0.04590522	-0.00001087	-0.00069011	0.0	0.0	0.0	0.33864737	-0.01577266
0.10	0.00355142	0.08603698	-0.00008639	-0.00269591	0.0	0.0	0.0	0.30083811	-0.03129824
0.15	0.00760022	0.11666542	-0.00028245	-0.00579988	0.0	0.0	0.0	0.24368131	-0.04041876
0.20	0.01261166	0.13511580	-0.00063985	-0.00968749	0.0	0.0	0.0	0.17797780	-0.04296431
0.25	0.01806253	0.13999635	-0.00118088	-0.01397419	0.0	0.0	0.0	0.11401129	-0.03969429
0.30	0.02341146	0.13130784	-0.00190750	-0.01824067	0.0	0.0	0.0	0.06014225	-0.03213107
0.35	0.02815489	0.11041665	-0.00280143	-0.02207470	0.0	0.0	0.0	0.02173984	-0.02229825
0.40	0.03187498	0.07988757	-0.00382637	-0.02511533	0.0	0.0	0.0	0.00060271	-0.01239494
0.45	0.03427541	0.04318721	-0.00493212	-0.02709328	0.0	0.0	0.0	-0.00504775	-0.00444914
0.50	0.03520292	0.00429056	-0.00606043	-0.02786094	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.21469080	-0.22220469	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.21778750	-0.22595024	-0.00038526	-0.00541185	0.00001980	0.00029604	0.00009952	0.00141325	-0.00000508
0.10	0.20151770	-0.20725977	-0.00036675	-0.00448032	0.00001958	0.00028204	0.00009455	0.00116328	-0.00000502
0.15	0.17118299	-0.17255187	-0.00031100	-0.00312973	0.00001822	0.00023972	0.00007984	0.00080685	-0.00000466
0.20	0.13301212	-0.12922817	-0.00022657	-0.00183359	0.00001508	0.00017529	0.00005785	0.00046932	-0.00000384
0.25	0.09329641	-0.08476734	-0.00013728	-0.00087705	0.00001055	0.00010670	0.00003485	0.00022297	-0.00000268
0.30	0.05752623	-0.04575821	-0.00006594	-0.00032406	0.00000589	0.00005151	0.00001665	0.00008188	-0.00000149
0.35	0.02962674	-0.01696948	-0.00002294	-0.00008289	0.00000239	0.00001800	0.00000577	0.00002084	-0.00000060
0.40	0.01141768	-0.00062569	-0.00000472	-0.00001146	0.00000057	0.00000372	0.00000119	0.00000288	-0.00000014
0.45	0.00240872	0.00392249	-0.00000030	-0.00000038	0.00000004	0.00000024	0.00000008	0.00000010	-0.00000001
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.18527496	0.02860519	-0.05453876	0.0	0.0
0.05	-0.00007646	-0.19852674	0.02433660	-0.04756061	0.0	0.0
0.10	-0.00007270	-0.19035929	0.01923619	-0.03830845	0.0	0.0
0.15	-0.00006153	-0.16579556	0.01402936	-0.02835745	0.0	0.0
0.20	-0.00004475	-0.13123316	0.00931602	-0.01905972	0.0	0.0
0.25	-0.00002709	-0.09339094	0.00550606	-0.01138038	0.0	0.0
0.30	-0.00001301	-0.05827951	0.00278602	-0.00580929	0.0	0.0
0.35	-0.00000453	-0.03032773	0.00112329	-0.00235958	0.0	0.0
0.40	-0.00000094	-0.01178634	0.00030936	-0.00065275	0.0	0.0
0.45	-0.00000006	-0.00248782	0.00003700	-0.00007758	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.00605488

Table 2

K= 0.10

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.69480264	0.01038918
0.05	0.00182367	0.09180975	-0.00002175	-0.00138124	0.0	0.0	0.0	0.67828798	-0.03079498
0.10	0.00709069	0.17205817	-0.00017309	-0.00540728	0.0	0.0	0.0	0.60309613	-0.06153391
0.15	0.01514288	0.23322314	-0.00056701	-0.01166990	0.0	0.0	0.0	0.48869169	-0.07954657
0.20	0.02505756	0.26983404	-0.00128764	-0.01956437	0.0	0.0	0.0	0.35677093	-0.08449233
0.25	0.03576593	0.27896672	-0.00238270	-0.02832499	0.0	0.0	0.0	0.22815585	-0.07788038
0.30	0.04617961	0.26053482	-0.00385855	-0.03708579	0.0	0.0	0.0	0.11989546	-0.06275904
0.35	0.05530784	0.21730971	-0.00567896	-0.04496758	0.0	0.0	0.0	0.04296845	-0.04320622
0.40	0.06235113	0.15462393	-0.00776871	-0.05118247	0.0	0.0	0.0	0.00096634	-0.02367059
0.45	0.06676376	0.07975745	-0.01002194	-0.05513710	0.0	0.0	0.0	-0.00998236	-0.00824682
0.50	0.06828612	0.00107408	-0.01231476	-0.05651266	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.42790478	-0.43161339	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.43464184	-0.43975985	-0.00147748	-0.02096935	0.00007676	0.00115416	0.00039571	0.00565786	-0.00002046
0.10	0.40296316	-0.40489823	-0.00140523	-0.01733209	0.00007590	0.00109944	0.00037582	0.00465639	-0.00002022
0.15	0.34311503	-0.33883601	-0.00118834	-0.01206911	0.00007060	0.00093404	0.00031690	0.00322671	-0.00001879
0.20	0.26718003	-0.25516319	-0.00086147	-0.00703485	0.00005836	0.00068213	0.00022896	0.00187232	-0.00001549
0.25	0.18760252	-0.16807157	-0.00051825	-0.00333886	0.00004073	0.00041416	0.00013729	0.00088509	-0.00001078
0.30	0.11554182	-0.09070724	-0.00024652	-0.00121941	0.00002262	0.00019896	0.00006511	0.00032215	-0.00000597
0.35	0.05921943	-0.03315122	-0.00008470	-0.00030648	0.00000910	0.00006901	0.00002233	0.00008078	-0.00000240
0.40	0.02258700	-0.00050939	-0.00001719	-0.00004123	0.00000217	0.00001412	0.00000454	0.00001088	-0.00000057
0.45	0.00469130	0.00822574	-0.00000110	-0.00000131	0.00000016	0.00000091	0.00000029	0.00000035	-0.00000004
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.36247468	0.05718996	-0.10661787	0.0	0.0
0.05	-0.00030905	-0.38881677	0.04871792	-0.09291333	0.0	0.0
0.10	-0.00029399	-0.37273502	0.03856053	-0.07472730	0.0	0.0
0.15	-0.00024906	-0.32422608	0.02815403	-0.05518347	0.0	0.0
0.20	-0.00018129	-0.25601697	0.01870104	-0.03695853	0.0	0.0
0.25	-0.00010971	-0.18147868	0.01104001	-0.02195581	0.0	0.0
0.30	-0.00005255	-0.11256373	0.00556688	-0.01112783	0.0	0.0
0.35	-0.00001819	-0.05803567	0.00222961	-0.00447511	0.0	0.0
0.40	-0.00000373	-0.02223847	0.00060771	-0.00122175	0.0	0.0
0.45	-0.00000024	-0.00460628	0.00007205	-0.00014357	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.02383659

Table 3

K= 0.15

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.04313660	0.01605700
0.05	0.00273434	0.13771391	-0.00003264	-0.00207339	0.0	0.0	0.0	1.01954365	-0.04518991
0.10	0.01061780	0.25806421	-0.00026008	-0.00813407	0.0	0.0	0.0	0.90726441	-0.09085035
0.15	0.02262740	0.34967381	-0.00085367	-0.01760953	0.0	0.0	0.0	0.73540950	-0.11753297
0.20	0.03733506	0.40415674	-0.00194330	-0.02962770	0.0	0.0	0.0	0.53664482	-0.12472707
0.25	0.05310335	0.41692054	-0.00360511	-0.04304270	0.0	0.0	0.0	0.34258360	-0.11468422
0.30	0.06829250	0.38771498	-0.00585203	-0.05651148	0.0	0.0	0.0	0.17931187	-0.09198791
0.35	0.08144540	0.32077312	-0.00862989	-0.06863141	0.0	0.0	0.0	0.06367981	-0.06280655
0.40	0.09142447	0.22441518	-0.01182149	-0.07812363	0.0	0.0	0.0	0.00106959	-0.03388901
0.45	0.09748846	0.11008751	-0.01525958	-0.08402210	0.0	0.0	0.0	-0.01481605	-0.01143078
0.50	0.09931892	-0.00906169	-0.01874767	-0.08582240	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.63965744	-0.62829971	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.65046448	-0.64136469	-0.00318792	-0.04572584	0.00016736	0.00253080	0.00088278	0.01271703	-0.00004623
0.10	0.60409534	-0.59269106	-0.00302919	-0.03773179	0.00016546	0.00241044	0.00083801	0.01046226	-0.00004570
0.15	0.51541078	-0.49847627	-0.00255427	-0.02618875	0.00015383	0.00204661	0.00070547	0.00724080	-0.00004245
0.20	0.40200824	-0.37731570	-0.00184220	-0.01518477	0.00012697	0.00149245	0.00050801	0.00418929	-0.00003500
0.25	0.28238362	-0.24937940	-0.00109990	-0.00714927	0.00008833	0.00090339	0.00030293	0.00196931	-0.00002433
0.30	0.17356586	-0.13436061	-0.00051787	-0.00257971	0.00004882	0.00043163	0.00014251	0.00070982	-0.00001343
0.35	0.08843166	-0.04819339	-0.00017562	-0.00063666	0.00001949	0.00014842	0.00004833	0.00017515	-0.00000536
0.40	0.03332841	0.00052340	-0.00003510	-0.00008322	0.00000460	0.00003001	0.00000969	0.00002297	-0.00000127
0.45	0.00679943	0.01294391	-0.00000221	-0.00000250	0.00000035	0.00000191	0.00000062	0.00000070	-0.00000010
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.53241938	0.08570737	-0.15644717	0.0	0.0
0.05	-0.00070071	-0.57164413	0.07308602	-0.13623387	0.0	0.0
0.10	-0.00066676	-0.54781777	0.05790835	-0.10939640	0.0	0.0
0.15	-0.00056522	-0.47587335	0.04230863	-0.08058298	0.0	0.0
0.20	-0.00041155	-0.37481350	0.02809610	-0.05377057	0.0	0.0
0.25	-0.00024881	-0.26460725	0.01655548	-0.03177458	0.0	0.0
0.30	-0.00011878	-0.16309136	0.00831202	-0.01598381	0.0	0.0
0.35	-0.00004085	-0.08327180	0.00330326	-0.00636033	0.0	0.0
0.40	-0.00000829	-0.03143109	0.00088971	-0.00171175	0.0	0.0
0.45	-0.00000054	-0.00637656	0.00010442	-0.00019861	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.05282111

Table 4

K= 0.20

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.39309311	0.02192925
0.05	0.00364424	0.18361753	-0.00004354	-0.00276655	0.0	0.0	0.0	1.36299706	-0.05906986
0.10	0.01413264	0.34405428	-0.00034737	-0.01087622	0.0	0.0	0.0	1.21381187	-0.11938113
0.15	0.03005311	0.46601683	-0.00114242	-0.02361811	0.0	0.0	0.0	0.98421073	-0.15452123
0.20	0.04944149	0.53808522	-0.00260674	-0.03987444	0.0	0.0	0.0	0.71787161	-0.16380841
0.25	0.07006806	0.55386883	-0.00484774	-0.05811748	0.0	0.0	0.0	0.45745534	-0.15023333
0.30	0.08973920	0.51289105	-0.00788682	-0.07649374	0.0	0.0	0.0	0.23845434	-0.11992693
0.35	0.10655785	0.42092013	-0.01165149	-0.09302062	0.0	0.0	0.0	0.08387661	-0.08118796
0.40	0.11909783	0.28949964	-0.01597928	-0.10586679	0.0	0.0	0.0	0.00089658	-0.04311834
0.45	0.12648124	0.13459361	-0.02063579	-0.11365151	0.0	0.0	0.0	-0.01955992	-0.01404255
0.50	0.12837863	-0.02549500	-0.02534523	-0.11567938	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.85004491	-0.81240010	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.86525178	-0.83078146	-0.00543694	-0.07883215	0.00028831	0.00438505	0.00155264	0.02254943	-0.00008230
0.10	0.80478191	-0.77051634	-0.00516124	-0.06493968	0.00028502	0.00417576	0.00147309	0.01854133	-0.00008135
0.15	0.68780291	-0.65121686	-0.00433901	-0.04492249	0.00026481	0.00354298	0.00123776	0.01281213	-0.00007556
0.20	0.53712100	-0.49532646	-0.00311269	-0.02590661	0.00021819	0.00257924	0.00088796	0.00738798	-0.00006227
0.25	0.37721753	-0.32828373	-0.00184380	-0.01209691	0.00015129	0.00155588	0.00052627	0.00345165	-0.00004320
0.30	0.23121262	-0.17634046	-0.00085882	-0.00431091	0.00008314	0.00073890	0.00024536	0.00123115	-0.00002376
0.35	0.11698717	-0.06182349	-0.00028720	-0.00104391	0.00003291	0.00025165	0.00008220	0.00029862	-0.00000942
0.40	0.04350216	0.00260303	-0.00005645	-0.00013241	0.00000768	0.00005021	0.00001624	0.00003807	-0.00000221
0.45	0.00869699	0.01809911	-0.00000352	-0.00000374	0.00000057	0.00000316	0.00000103	0.00000110	-0.00000017
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.69590771	0.11412895	-0.20423603	0.0	0.0
0.05	-0.00125220	-0.74777204	0.09740090	-0.17770034	0.0	0.0
0.10	-0.00119181	-0.71629870	0.07723147	-0.14245951	0.0	0.0
0.15	-0.00101070	-0.62132972	0.05644281	-0.10466611	0.0	0.0
0.20	-0.00073580	-0.48810208	0.03745479	-0.06957448	0.0	0.0
0.25	-0.00044410	-0.34314120	0.02201564	-0.04088864	0.0	0.0
0.30	-0.00021111	-0.21011883	0.01099722	-0.02040784	0.0	0.0
0.35	-0.00007203	-0.10619813	0.00433195	-0.00803031	0.0	0.0
0.40	-0.00001445	-0.03944653	0.00115126	-0.00212797	0.0	0.0
0.45	-0.00000093	-0.00782198	0.00013354	-0.00024344	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.09254831

Table 5

K= 0.25

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.74540806	0.02793863
0.05	0.00455335	0.22952074	-0.00005445	-0.00346072	0.0	0.0	0.0	1.70924664	-0.07253546
0.10	0.01763518	0.43002927	-0.00043496	-0.01363371	0.0	0.0	0.0	1.52323151	-0.14725107
0.15	0.03741943	0.58225304	-0.00143325	-0.02969511	0.0	0.0	0.0	1.23548698	-0.19064814
0.20	0.06137440	0.67162281	-0.00327788	-0.05030164	0.0	0.0	0.0	0.90074670	-0.20187354
0.25	0.08665401	0.68982673	-0.00611021	-0.07353950	0.0	0.0	0.0	0.57295692	-0.18465453
0.30	0.11051035	0.63611245	-0.00996181	-0.09700972	0.0	0.0	0.0	0.29741079	-0.14668715
0.35	0.13063693	0.51788002	-0.01474113	-0.11809152	0.0	0.0	0.0	0.10358328	-0.09844434
0.40	0.14537644	0.35014659	-0.02023686	-0.13434410	0.0	0.0	0.0	0.00044498	-0.05143248
0.45	0.15377748	0.15373629	-0.02614180	-0.14393800	0.0	0.0	0.0	-0.02422102	-0.01612781
0.50	0.15554577	-0.04756214	-0.03209458	-0.14598966	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.05922794	-0.98409200	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.07908440	-1.00809097	-0.00815449	-0.11954194	0.00043662	0.00667959	0.00239582	0.03509826	-0.00012846
0.10	1.00499344	-0.93833995	-0.00773317	-0.09830385	0.00043159	0.00635937	0.00227166	0.02883971	-0.00012697
0.15	0.86013556	-0.79690939	-0.00648104	-0.06777072	0.00040070	0.00539136	0.00190472	0.01989201	-0.00011792
0.20	0.67225540	-0.60895377	-0.00462359	-0.03886817	0.00032956	0.00391733	0.00136078	0.01142820	-0.00009709
0.25	0.47178781	-0.40449399	-0.00271628	-0.01799631	0.00022769	0.00235412	0.00080113	0.00530396	-0.00006720
0.30	0.28818774	-0.21637249	-0.00125098	-0.00633151	0.00012437	0.00111059	0.00036989	0.00187093	-0.00003679
0.35	0.14467883	-0.07384843	-0.00041219	-0.00150334	0.00004876	0.00037427	0.00012228	0.00044566	-0.00001447
0.40	0.05300861	0.00581443	-0.00007958	-0.00018478	0.00001123	0.00007360	0.00002377	0.00005514	-0.00000335
0.45	0.01036014	0.02370055	-0.00000490	-0.00000490	0.00000083	0.00000458	0.00000149	0.00000149	-0.00000025
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.85371143	0.14244193	-0.25019354	0.0	0.0
0.05	-0.00196275	-0.91794980	0.12163943	-0.21749252	0.0	0.0
0.10	-0.00186839	-0.87886626	0.09649837	-0.17406398	0.0	0.0
0.15	-0.00158471	-0.76119387	0.07052058	-0.12754667	0.0	0.0
0.20	-0.00115305	-0.59637433	0.04674280	-0.08445293	0.0	0.0
0.25	-0.00069437	-0.41746032	0.02739292	-0.04935335	0.0	0.0
0.30	-0.00032839	-0.25391901	0.01360448	-0.02443305	0.0	0.0
0.35	-0.00011103	-0.12698972	0.00530683	-0.00950152	0.0	0.0
0.40	-0.00002198	-0.04637499	0.00138955	-0.00247615	0.0	0.0
0.45	-0.00000140	-0.00896831	0.00015906	-0.00027886	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.14261717

Table 6

K= 0.30

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.10075474	0.03402933
0.05	0.00546168	0.27542323	-0.00006537	-0.00415590	0.0	0.0	0.0	2.05885410	-0.08567446
0.10	0.02112532	0.51598835	-0.00052285	-0.01640642	0.0	0.0	0.0	1.83600140	-0.17457235
0.15	0.04472574	0.69838238	-0.00172614	-0.03583987	0.0	0.0	0.0	1.48965359	-0.22604012
0.20	0.07313108	0.80477226	-0.00395662	-0.06090624	0.0	0.0	0.0	1.08559513	-0.23905212
0.25	0.10285521	0.82481009	-0.00739216	-0.08929902	0.0	0.0	0.0	0.68930978	-0.21807164
0.30	0.13059676	0.75743657	-0.01207590	-0.11803615	0.0	0.0	0.0	0.35630280	-0.17238045
0.35	0.15367663	0.61180121	-0.01789617	-0.14380169	0.0	0.0	0.0	0.12285030	-0.11467260
0.40	0.17026979	0.40665549	-0.02458916	-0.16349244	0.0	0.0	0.0	-0.00027238	-0.05890999
0.45	0.17941755	0.16800958	-0.03176932	-0.17480475	0.0	0.0	0.0	-0.02880226	-0.01773541
0.50	0.18090528	-0.07457960	-0.03898404	-0.17667717	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.26743507	-1.14358234	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.29211903	-1.17341137	-0.01127989	-0.16721159	0.00060964	0.00938119	0.00340224	0.05029937	-0.00018443
0.10	1.20477676	-1.09618664	-0.01068600	-0.13726199	0.00060253	0.00892929	0.00322371	0.04129644	-0.00018230
0.15	1.03234673	-0.93548918	-0.00892716	-0.09430176	0.00055900	0.00756333	0.00269677	0.02842588	-0.00016926
0.20	0.80724972	-0.71805698	-0.00633224	-0.05378266	0.00045885	0.00548396	0.00191803	0.01626593	-0.00013920
0.25	0.56587791	-0.47782636	-0.00368841	-0.02468836	0.00031581	0.00328207	0.00112112	0.00749636	-0.00009609
0.30	0.34428489	-0.25427985	-0.00167876	-0.00857269	0.00017138	0.00153729	0.00051223	0.00261362	-0.00005232
0.35	0.17136544	-0.08414930	-0.00054456	-0.00199458	0.00006650	0.00051218	0.00016692	0.00061088	-0.00002039
0.40	0.06178576	0.01020044	-0.00010314	-0.00023724	0.00001510	0.00009912	0.00003188	0.00007326	-0.00000467
0.45	0.01177637	0.02974602	-0.00000626	-0.00000586	0.00000111	0.00000610	0.00000198	0.00000186	-0.00000035
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.00654793	0.17064756	-0.29452157	0.0	0.0
0.05	-0.00283054	-1.08288574	0.14579177	-0.25578690	0.0	0.0
0.10	-0.00269469	-1.03618431	0.11569077	-0.20435619	0.0	0.0
0.15	-0.00228546	-0.89605492	0.08451915	-0.14933985	0.0	0.0
0.20	-0.00166143	-0.70012349	0.05593712	-0.09849089	0.0	0.0
0.25	-0.00099774	-0.48795342	0.03266851	-0.05722647	0.0	0.0
0.30	-0.00046910	-0.29477614	0.01612173	-0.02809439	0.0	0.0
0.35	-0.00015701	-0.14583206	0.00622225	-0.01079147	0.0	0.0
0.40	-0.00003064	-0.05231289	0.00160296	-0.00276244	0.0	0.0
0.45	-0.00000193	-0.00984311	0.00018079	-0.00030575	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.20268005

Table 7

K= 0.35

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.45979118	0.04015847
0.05	0.00636924	0.32132548	-0.00007630	-0.00485210	0.0	0.0	0.0	2.41238403	-0.09856117
0.10	0.02460303	0.60193217	-0.00061103	-0.01919434	0.0	0.0	0.0	2.15262794	-0.20144445
0.15	0.05197148	0.81440556	-0.00202108	-0.04205182	0.0	0.0	0.0	1.74715900	-0.26081318
0.20	0.08470947	0.93753725	-0.00464288	-0.07168514	0.0	0.0	0.0	1.27278328	-0.27546650
0.25	0.11866629	0.95883673	-0.00869324	-0.10538638	0.0	0.0	0.0	0.80677468	-0.25060409
0.30	0.14999092	0.87692612	-0.01422797	-0.13955021	0.0	0.0	0.0	0.41528749	-0.19711751
0.35	0.17567319	0.70284492	-0.02111405	-0.17010981	0.0	0.0	0.0	0.14175653	-0.12997180
0.40	0.19378972	0.45934850	-0.02903132	-0.19325340	0.0	0.0	0.0	-0.00122565	-0.06563234
0.45	0.20344400	0.17793161	-0.03751064	-0.20618528	0.0	0.0	0.0	-0.03330137	-0.01891629
0.50	0.20454079	-0.10585290	-0.04600310	-0.20768255	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.47492123	-1.29107380	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.50455570	-1.32688236	-0.01476182	-0.22130054	0.00080506	0.01246126	0.00456187	0.06808811	-0.00024991
0.10	1.40425682	-1.24412537	-0.01396976	-0.18134069	0.00079557	0.01185787	0.00431931	0.05584987	-0.00024700
0.15	1.20445442	-1.06695461	-0.01163227	-0.12415189	0.00073752	0.01003426	0.00360439	0.03835846	-0.00022925
0.20	0.94203126	-0.82257700	-0.00820263	-0.07040864	0.00060415	0.00725923	0.00255133	0.02185681	-0.00018830
0.25	0.65935892	-0.54818583	-0.00473592	-0.03204005	0.00041414	0.00432554	0.00147996	0.00999928	-0.00012958
0.30	0.39937544	-0.28996998	-0.00212925	-0.01097789	0.00022318	0.00201058	0.00066871	0.00344430	-0.00007015
0.35	0.19696522	-0.09267318	-0.00067944	-0.00250151	0.00008566	0.00066169	0.00021461	0.00078936	-0.00002707
0.40	0.06980580	0.01576627	-0.00012607	-0.00028754	0.00001915	0.00012584	0.00004021	0.00009162	-0.00000611
0.45	0.01294335	0.03622312	-0.00000754	-0.00000659	0.00000139	0.00000764	0.00000247	0.00000216	-0.00000046
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.15511322	0.19875854	-0.33741885	0.0	0.0
0.05	-0.00385324	-1.24326992	0.16986126	-0.29275739	0.0	0.0
0.10	-0.00366844	-1.18889904	0.13480335	-0.23348171	0.0	0.0
0.15	-0.00311068	-1.02648926	0.09842730	-0.17016131	0.0	0.0
0.20	-0.00225861	-0.79983950	0.06502473	-0.11177438	0.0	0.0
0.25	-0.00135197	-0.55501205	0.03783151	-0.06456697	0.0	0.0
0.30	-0.00063151	-0.33298081	0.01854226	-0.03142793	0.0	0.0
0.35	-0.00020902	-0.16291678	0.00707552	-0.01191841	0.0	0.0
0.40	-0.00004014	-0.05736077	0.00179094	-0.00299326	0.0	0.0
0.45	-0.00000250	-0.01047527	0.00019868	-0.00032501	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.27243674

Table 8

K= 0.40

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.82315254	0.04629330
0.05	0.00727602	0.36722726	-0.00008724	-0.00554931	0.0	0.0	0.0	2.77039433	-0.11125827
0.10	0.02806820	0.68786055	-0.00069951	-0.02199741	0.0	0.0	0.0	2.47362041	-0.22795469
0.15	0.05915599	0.93032283	-0.00231807	-0.04833036	0.0	0.0	0.0	2.00846577	-0.29507166
0.20	0.09610713	1.06991863	-0.00533657	-0.08263528	0.0	0.0	0.0	1.46270847	-0.31122881
0.25	0.13408160	1.09192085	-0.01001307	-0.12179154	0.0	0.0	0.0	0.92565209	-0.28236395
0.30	0.16868520	0.99464470	-0.01641691	-0.16152900	0.0	0.0	0.0	0.47456187	-0.22100550
0.35	0.19662368	0.79118359	-0.02439217	-0.19697660	0.0	0.0	0.0	0.16041297	-0.14444071
0.40	0.21595001	0.50856650	-0.03355857	-0.22357345	0.0	0.0	0.0	-0.00236611	-0.07168311
0.45	0.22590029	0.18404067	-0.04335847	-0.23802257	0.0	0.0	0.0	-0.03771056	-0.01972321
0.50	0.22653443	-0.14068300	-0.05314228	-0.23896223	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.68196487	-1.42673302	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.71663857	-1.46862888	-0.01855784	-0.28136617	0.00102101	0.01589627	0.00586535	0.08840513	-0.00032458
0.10	1.60360813	-1.38223457	-0.01754313	-0.23015231	0.00100885	0.01512238	0.00554908	0.07244241	-0.00032079
0.15	1.37654400	-1.19133949	-0.01455910	-0.15702319	0.00093448	0.01278377	0.00461854	0.04963768	-0.00029760
0.20	1.07659626	-0.92251098	-0.01020507	-0.08854902	0.00076386	0.00922653	0.00325279	0.02815899	-0.00024408
0.25	0.75217605	-0.61554825	-0.00583903	-0.03994385	0.00052143	0.00547251	0.00187180	0.01278510	-0.00016740
0.30	0.45340055	-0.32342201	-0.00259220	-0.01350250	0.00027896	0.00252329	0.00083595	0.00434939	-0.00009004
0.35	0.22145081	-0.09942585	-0.00081308	-0.00301202	0.00010581	0.00081964	0.00026398	0.00097680	-0.00003437
0.40	0.07707226	0.02248277	-0.00014759	-0.00033419	0.00002325	0.00015296	0.00004846	0.00010960	-0.00000764
0.45	0.01386807	0.04311031	-0.00000869	-0.00000708	0.00000167	0.00000916	0.00000294	0.00000240	-0.00000056
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.30002308	0.22679687	-0.37906992	0.0	0.0
0.05	-0.00502843	-1.39972591	0.19386095	-0.32856899	0.0	0.0
0.10	-0.00478723	-1.33761311	0.15384108	-0.26158088	0.0	0.0
0.15	-0.00405796	-1.15304661	0.11224377	-0.19012386	0.0	0.0
0.20	-0.00294215	-0.89599758	0.07400137	-0.12438905	0.0	0.0
0.25	-0.00175474	-0.61902350	0.04287794	-0.07143450	0.0	0.0
0.30	-0.00081383	-0.36882585	0.02086421	-0.03447056	0.0	0.0
0.35	-0.00026613	-0.17844039	0.00786658	-0.01290115	0.0	0.0
0.40	-0.00005023	-0.06162247	0.00195395	-0.00317527	0.0	0.0
0.45	-0.00000310	-0.01089471	0.00021285	-0.00033759	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.35162604

Table 9

K= 0.45

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.19147587	0.05241552
0.05	0.00818201	0.41312838	-0.00009819	-0.00624753	0.0	0.0	0.0	3.13347340	-0.12381339
0.10	0.03152082	0.77377319	-0.00078828	-0.02481555	0.0	0.0	0.0	2.79954624	-0.25417697
0.15	0.06627870	1.04613304	-0.00261708	-0.05467485	0.0	0.0	0.0	2.27411842	-0.32890922
0.20	0.10732162	1.20192432	-0.00603760	-0.09375346	0.0	0.0	0.0	1.65585136	-0.34644526
0.25	0.14909619	1.22409534	-0.01135125	-0.13850480	0.0	0.0	0.0	1.04631996	-0.31346184
0.30	0.18667358	1.11068344	-0.01864162	-0.18395114	0.0	0.0	0.0	0.53438735	-0.24415314
0.35	0.21652782	0.87701792	-0.02772814	-0.22436535	0.0	0.0	0.0	0.17897594	-0.15818167
0.40	0.23676705	0.55468160	-0.03816658	-0.25440490	0.0	0.0	0.0	-0.00362034	-0.07714969
0.45	0.24683076	0.18689829	-0.04930635	-0.27027076	0.0	0.0	0.0	-0.04201495	-0.02021103
0.50	0.24696583	-0.17836875	-0.06039353	-0.27048832	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.88888550	-1.55069828	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.92866135	-1.59875870	-0.02263463	-0.34707075	0.00125604	0.01966850	0.00730439	0.11120427	-0.00040818
0.10	1.80306816	-1.51060104	-0.02137365	-0.28340024	0.00124091	0.01870566	0.00690481	0.09102809	-0.00040338
0.15	1.54878521	-1.30872059	-0.01767835	-0.19268787	0.00114850	0.01579645	0.00573129	0.06222029	-0.00037403
0.20	1.21103668	-1.01792145	-0.01231628	-0.10805380	0.00093677	0.01137316	0.00401542	0.03513705	-0.00030628
0.25	0.84436315	-0.67996663	-0.00698248	-0.04831912	0.00063671	0.00671368	0.00229150	0.01583039	-0.00020930
0.30	0.50637764	-0.35469139	-0.00305988	-0.01611412	0.00033806	0.00306974	0.00101103	0.00531757	-0.00011181
0.35	0.24485165	-0.10447347	-0.00094283	-0.00351795	0.00012665	0.00098356	0.00031391	0.00116980	-0.00004220
0.40	0.08362019	0.03028742	-0.00016721	-0.00037638	0.00002733	0.00017986	0.00005639	0.00012676	-0.00000922
0.45	0.01456639	0.05037779	-0.00000968	-0.00000733	0.00000193	0.00001060	0.00000338	0.00000256	-0.00000067
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.44185257	0.25479549	-0.41965586	0.0	0.0
0.05	-0.00635437	-1.55285454	0.21781600	-0.36338568	0.0	0.0
0.10	-0.00604927	-1.48291492	0.17282063	-0.28879499	0.0	0.0
0.15	-0.00512540	-1.27627468	0.12597853	-0.20934278	0.0	0.0
0.20	-0.00370994	-0.98907745	0.08287275	-0.13642257	0.0	0.0
0.25	-0.00220397	-0.68038410	0.04781152	-0.07789069	0.0	0.0
0.30	-0.00101445	-0.40261418	0.02309079	-0.03726060	0.0	0.0
0.35	-0.00032750	-0.19260722	0.00859813	-0.01375928	0.0	0.0
0.40	-0.00006066	-0.06520653	0.00209339	-0.00331541	0.0	0.0
0.45	-0.00000370	-0.01113262	0.00022354	-0.00034445	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.44002986

Table 10

K= 0.50

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.56540203	0.05851371
0.05	0.00908722	0.45902908	-0.00010915	-0.00694676	0.0	0.0	0.0	3.50221825	-0.13626623
0.10	0.03496078	0.85967046	-0.00087735	-0.02764873	0.0	0.0	0.0	3.13098717	-0.28017581
0.15	0.07333899	1.16183853	-0.00291810	-0.06108467	0.0	0.0	0.0	2.54468346	-0.36240995
0.20	0.11835080	1.33355904	-0.00674589	-0.10503703	0.0	0.0	0.0	1.85272980	-0.38121182
0.25	0.16370517	1.35538197	-0.01270747	-0.15551674	0.0	0.0	0.0	1.16919994	-0.34399903
0.30	0.20395041	1.22511673	-0.02090107	-0.20679516	0.0	0.0	0.0	0.59506571	-0.26666272
0.35	0.23538536	0.96054667	-0.03111959	-0.25224113	0.0	0.0	0.0	0.19763494	-0.17129397
0.40	0.25625694	0.59807038	-0.04285116	-0.28570491	0.0	0.0	0.0	-0.00489576	-0.08211905
0.45	0.26627803	0.18706930	-0.05534825	-0.30289280	0.0	0.0	0.0	-0.04619375	-0.02043473
0.50	0.26590919	-0.21821934	-0.06774956	-0.30224538	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	2.09599400	-1.66305637	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	2.14092445	-1.71733952	-0.02696686	-0.41816783	0.00150915	0.02376573	0.00887214	0.13645256	-0.00050051
0.10	2.00289822	-1.62928963	-0.02543660	-0.34086812	0.00149077	0.02259592	0.00837962	0.11157280	-0.00049460
0.15	1.72137451	-1.41916275	-0.02096770	-0.23097968	0.00137862	0.01906169	0.00693592	0.07607263	-0.00045837
0.20	1.34548569	-1.10888481	-0.01451869	-0.12881434	0.00112202	0.01369026	0.00483328	0.04276246	-0.00037471
0.25	0.93600482	-0.74152941	-0.00815492	-0.05710840	0.00075929	0.00804238	0.00273472	0.01911612	-0.00025509
0.30	0.55837470	-0.38387829	-0.00352688	-0.01879079	0.00040005	0.00364583	0.00119158	0.00633987	-0.00013531
0.35	0.26723731	-0.10792249	-0.00106699	-0.00401444	0.00014796	0.00115163	0.00036354	0.00136585	-0.00005045
0.40	0.08950764	0.03909308	-0.00018465	-0.00041383	0.00003130	0.00020611	0.00006383	0.00014283	-0.00001083
0.45	0.01506090	0.05798938	-0.00001049	-0.00000737	0.00000218	0.00001194	0.00000377	0.00000265	-0.00000078
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.58113384	0.28279251	-0.45934564	0.0	0.0
0.05	-0.00783002	-1.70320606	0.24175715	-0.39736140	0.0	0.0
0.10	-0.00745351	-1.62535095	0.19176537	-0.31525761	0.0	0.0
0.15	-0.00631176	-1.39668465	0.13964820	-0.22792792	0.0	0.0
0.20	-0.00456046	-1.07953262	0.09165043	-0.14795995	0.0	0.0
0.25	-0.00269797	-0.73947513	0.05264093	-0.08399594	0.0	0.0
0.30	-0.00123202	-0.43464160	0.02522879	-0.03983597	0.0	0.0
0.35	-0.00039242	-0.20561999	0.00927481	-0.01451235	0.0	0.0
0.40	-0.00007125	-0.06822145	0.00221140	-0.00342062	0.0	0.0
0.45	-0.00000429	-0.01122032	0.00023111	-0.00034659	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.53745592

Table 11

K= 0.0

A1	A2	A3	A4
0.0	0.0	0.0	0.0

B1	B2	B3	B4
0.0	0.0	0.0	0.0

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0
0.30	0.0	0.0	0.0	0.0	0.0	0.0
0.35	0.0	0.0	0.0	0.0	0.0	0.0
0.40	0.0	0.0	0.0	0.0	0.0	0.0
0.45	0.0	0.0	0.0	0.0	0.0	0.0
0.50	0.0	0.0	0.0	0.0	0.0	0.0

J(CST)= 0.0

Table 12

K= 0.05

A1	A2	A3	A4
-0.00098153	-0.01320701	0.0	0.00075508

B1	B2	B3	B4
0.00025427	0.00346635	0.0	-0.00019557

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.34225446	0.00526662
0.05	0.00091221	0.04590390	-0.00001057	-0.00069011	-0.00000435	0.00001833	-0.00013191	0.33389539	-0.01555482
0.10	0.00355110	0.08601683	-0.00008414	-0.00269566	-0.00003205	0.00006601	-0.00047503	0.29640639	-0.03108425
0.15	0.00759795	0.11657321	-0.00027548	-0.00579813	-0.00009703	0.00013069	-0.00094049	0.23961240	-0.04016151
0.20	0.01260293	0.13485742	-0.00062488	-0.00968067	-0.00020328	0.00020023	-0.00144090	0.17438132	-0.04262619
0.25	0.01803855	0.13944751	-0.00115453	-0.01395519	-0.00034621	0.00026353	-0.00189648	0.11103451	-0.03926331
0.30	0.02335852	0.13033706	-0.00186663	-0.01819815	-0.00051456	0.00031138	-0.00224093	0.05791880	-0.03163076
0.35	0.02805471	0.10891587	-0.00274330	-0.02199324	-0.00069259	0.00033711	-0.00242622	0.02032950	-0.02178855
0.40	0.03170615	0.07780272	-0.00374872	-0.02497669	-0.00086243	0.00033705	-0.00242594	-0.00006174	-0.01196303
0.45	0.03401581	0.04054351	-0.00483319	-0.02687877	-0.00100651	0.00031071	-0.00223655	-0.00519100	-0.00419161
0.50	0.03483288	0.00120692	-0.00593889	-0.02755445	-0.00110969	0.00026064	-0.00187641	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.21183127	-0.21907341	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.21506113	-0.22295982	-0.00035306	-0.00514495	0.00001696	0.00027104	0.00009112	0.00134299	-0.00000434
0.10	0.19890434	-0.20437878	-0.00033499	-0.00423168	0.00001675	0.00025736	0.00008627	0.00109807	-0.00000429
0.15	0.16870928	-0.16980803	-0.00028070	-0.00291573	0.00001546	0.00021616	0.00007196	0.00075099	-0.00000395
0.20	0.13075382	-0.12671131	-0.00019926	-0.00166607	0.00001251	0.00015401	0.00005076	0.00042579	-0.00000318
0.25	0.09136182	-0.08260906	-0.00011456	-0.00076001	0.00000830	0.00008895	0.00002897	0.00019265	-0.00000209
0.30	0.05602431	-0.04409286	-0.00004889	-0.00025290	0.00000409	0.00003815	0.00001224	0.00006347	-0.00000102
0.35	0.02862607	-0.01588218	-0.00001179	-0.00004662	0.00000112	0.00000924	0.00000288	0.00001145	-0.00000027
0.40	0.01090639	-0.00010056	0.00000124	0.00000301	-0.00000016	-0.00000098	-0.00000036	-0.00000088	0.00000005
0.45	0.00226777	0.00403795	0.00000183	0.00000339	-0.00000025	-0.00000145	-0.00000048	-0.00000089	0.00000006
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.18293083	0.02819414	-0.05379499	0.0	0.0
0.05	-0.00006994	-0.19622582	0.02396748	-0.04687459	-0.00060767	0.00015949
0.10	-0.00006627	-0.18807536	0.01891411	-0.03769367	-0.00114155	0.00029959
0.15	-0.00005540	-0.16355479	0.01376083	-0.02783167	-0.00155142	0.00040715
0.20	-0.00003923	-0.12911880	0.00910652	-0.01863960	-0.00180074	0.00047256
0.25	-0.00002249	-0.09152716	0.00535728	-0.01107511	-0.00186993	0.00049069
0.30	-0.00000955	-0.05679701	0.00269375	-0.00561542	-0.00175803	0.00046129
0.35	-0.00000226	-0.02931894	0.00107685	-0.00225908	-0.00148260	0.00038897
0.40	0.00000028	-0.01126090	0.00029316	-0.00061583	-0.00107752	0.00028264
0.45	0.00000038	-0.00234030	0.00003455	-0.00007095	-0.00058914	0.00015444
0.50	0.0	0.0	0.0	0.0	-0.00007094	0.00001843

J(CST)= 0.00597097

Table 13

K= 0.10

A1	A2	A3	A4
-0.01227599	-0.24222934	0.00028183	0.00942041

B1	B2	B3	B4
0.00317945	0.06508023	0.0	-0.00244745

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.57895929	0.01469296
0.05	0.00182330	0.09176129	-0.00001046	-0.00138096	-0.00015959	0.00068786	-0.00483540	0.56559956	-0.03013899
0.10	0.00707899	0.17132264	-0.00008860	-0.00539827	-0.00117383	0.00247183	-0.01737678	0.49013007	-0.06203527
0.15	0.01506017	0.22984672	-0.00030659	-0.01160491	-0.00354475	0.00486970	-0.03423523	0.37628996	-0.07840520
0.20	0.02474121	0.26038998	-0.00073040	-0.01930935	-0.00739162	0.00739028	-0.05195829	0.24982882	-0.07944900
0.25	0.03490524	0.25900388	-0.00140848	-0.02761182	-0.01249437	0.00957085	-0.06729299	0.13412637	-0.06793630
0.30	0.04429900	0.22550911	-0.00236188	-0.03548605	-0.01836029	0.01102004	-0.07748801	0.04633398	-0.04858692
0.35	0.05178791	0.16379422	-0.00357750	-0.04190613	-0.02431377	0.01145927	-0.08058441	-0.00529152	-0.02713885
0.40	0.05648759	0.08152258	-0.00500678	-0.04600137	-0.02960265	0.01075516	-0.07564396	-0.02231957	-0.00924210
0.45	0.05785856	-0.01076214	-0.00657065	-0.04720584	-0.03351157	0.00893622	-0.06286585	-0.01510885	0.00062473
0.50	0.05576067	-0.10102481	-0.00817017	-0.04536432	-0.03546884	0.00618916	-0.04356229	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.36588001	-0.36932737	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.37254578	-0.37697792	0.00023083	-0.00588209	-0.00003152	-0.00019948	-0.00006329	0.00154472	0.00000835
0.10	0.33817697	-0.33804643	0.00028291	-0.00322339	-0.00003180	-0.00023891	-0.00007733	0.00082862	0.00000843
0.15	0.27579200	-0.26773620	0.00041461	-0.00003060	-0.00003343	-0.00033934	-0.00011230	-0.00001884	0.00000887
0.20	0.20030767	-0.18314946	0.00055314	0.00207806	-0.00003625	-0.00044609	-0.00014851	-0.00056973	0.00000962
0.25	0.12624961	-0.10116434	0.00060066	0.00256695	-0.00003766	-0.00048320	-0.00016020	-0.00068964	0.00000997
0.30	0.06536776	-0.03584835	0.00051048	0.00188268	-0.00003404	-0.00041189	-0.00013546	-0.00050192	0.00000899
0.35	0.02452942	0.00424114	0.00031807	0.00088520	-0.00002382	-0.00025780	-0.00008400	-0.00023505	0.00000626
0.40	0.00443225	0.01804153	0.00011528	0.00022347	-0.00000987	-0.00009375	-0.00003016	-0.00005928	0.00000256
0.45	-0.00037503	0.01233531	-0.00000626	0.00002562	0.00000082	0.00000517	0.00000192	-0.00000692	-0.00000025
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.31801140	0.04660171	-0.08889049	0.0	0.0
0.05	0.00005454	-0.34251320	0.03863079	-0.07536632	-0.02226267	0.00598102
0.10	0.00006517	-0.32131433	0.02920658	-0.05784240	-0.04163714	0.01118543
0.15	0.00009183	-0.26742995	0.01990694	-0.03980440	-0.05596990	0.01503476
0.20	0.00011973	-0.19677985	0.01195230	-0.02402066	-0.06355977	0.01707216
0.25	0.00012886	-0.12512738	0.00605909	-0.01217288	-0.06342721	0.01703458
0.30	0.00010930	-0.06521726	0.00238605	-0.00473180	-0.05550369	0.01490388
0.35	0.00006808	-0.02457905	0.00059553	-0.00109672	-0.04070729	0.01092698
0.40	0.00002453	-0.00437815	0.00003149	0.00002592	-0.02087536	0.00559769
0.45	-0.00000158	0.00047752	-0.00001385	0.00007264	0.00145008	-0.00040091
0.50	0.0	0.0	0.0	0.0	0.02335700	-0.00628640

J(CST)= 0.02060221

Table 14

K= 0.15

A1	A2	A3	A4
-0.01243632	-0.28304714	0.00028183	0.00942041

B1	B2	B3	B4
0.00317945	0.07639152	0.0	-0.00244745

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.86143011	0.02133010
0.05	0.00273369	0.13762885	-0.00001277	-0.00207289	-0.00027968	0.00121089	-0.00847375	0.83922738	-0.04714910
0.10	0.01059735	0.25677454	-0.00011138	-0.00811819	-0.00205666	0.00434933	-0.03043792	0.72148454	-0.09519517
0.15	0.02248348	0.34375554	-0.00039556	-0.01749422	-0.00620732	0.00855940	-0.05990440	0.54560423	-0.11888391
0.20	0.03678774	0.38761526	-0.00096384	-0.02917219	-0.01293068	0.01296262	-0.09072626	0.35211301	-0.11874193
0.25	0.05162437	0.38200682	-0.00189479	-0.04176075	-0.02182057	0.01672538	-0.11707026	0.17767435	-0.09938461
0.30	0.06508476	0.32661694	-0.00322861	-0.05362183	-0.03198194	0.01913999	-0.13398319	0.04878161	-0.06851119
0.35	0.07548517	0.22780263	-0.00495347	-0.06308711	-0.04219176	0.01970576	-0.13796014	-0.02267746	-0.03536800
0.40	0.08156019	0.09817529	-0.00700083	-0.06874186	-0.05109501	0.01819436	-0.12740129	-0.04090499	-0.00890728
0.45	0.08258510	-0.04492842	-0.00925246	-0.06970519	-0.05741629	0.01468406	-0.10285264	-0.02421970	0.00401419
0.50	0.07843739	-0.18188614	-0.01155806	-0.06582248	-0.06016040	0.00955246	-0.06695497	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.54507065	-0.54136634	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.55344468	-0.55090582	0.00095826	-0.00813664	-0.00007715	-0.00082562	-0.00025192	0.00223904	0.00002035
0.10	0.49930304	-0.49116707	0.00106897	-0.00252821	-0.00007765	-0.00090958	-0.00028222	0.00070555	0.00002049
0.15	0.40281755	-0.38480777	0.00133043	0.00380858	-0.00008038	-0.00110987	-0.00035303	-0.00101031	0.00002124
0.20	0.28706080	-0.25723839	0.00156307	0.00735620	-0.00008447	-0.00129092	-0.00041566	-0.00196509	0.00002234
0.25	0.17469490	-0.13415480	0.00155017	0.00720516	-0.00008413	-0.00128074	-0.00041220	-0.00192430	0.00002226
0.30	0.08415788	-0.03760980	0.00121362	0.00469566	-0.00007180	-0.00100855	-0.00032214	-0.00125261	0.00001897
0.35	0.02599280	0.01884716	0.00067291	0.00196012	-0.00004501	-0.00056315	-0.00017779	-0.00052214	0.00001182
0.40	0.00047166	0.03411735	0.00016726	0.00040959	-0.00001188	-0.00014065	-0.00004316	-0.00010919	0.00000301
0.45	-0.00238350	0.02042531	-0.00008724	0.00007644	0.00000989	0.00007374	0.00002413	-0.00002094	-0.00000274
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.47074300	0.06833291	-0.12890834	0.0	0.0
0.05	0.00021734	-0.50510710	0.05618199	-0.10820937	-0.03900897	0.01052744
0.10	0.00024030	-0.46974963	0.04191960	-0.08174956	-0.07288754	0.01966896
0.15	0.00029455	-0.38539612	0.02796992	-0.05489686	-0.09774339	0.02637431
0.20	0.00034328	-0.27713758	0.01619857	-0.03183843	-0.11044586	0.02979888
0.25	0.00034055	-0.16952842	0.00767935	-0.01501432	-0.10916179	0.02944842
0.30	0.00026771	-0.08201218	0.00259937	-0.00494821	-0.09376329	0.02528893
0.35	0.00014879	-0.02545014	0.00035429	-0.00050791	-0.06601322	0.01779659
0.40	0.00003630	-0.00040263	-0.00015175	0.00045035	-0.02945209	0.00792731
0.45	-0.00002039	0.00247660	-0.00004994	0.00017206	0.01103054	-0.00299897
0.50	0.0	0.0	0.0	0.0	0.04988354	-0.01348407

J(CST)= 0.04585382

Table 15

K= 0.20

A1	A2	A3	A4
0.01633753	-0.41949201	-0.00091682	-0.01575423

B1	B2	B3	B4
-0.00453819	0.11382747	0.0	0.00430573

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.14647388	0.02329922
0.05	0.00364296	0.18344969	-0.00000410	-0.00276557	-0.00055191	0.00240116	-0.01671256	1.10324669	-0.07716322
0.10	0.01409244	0.34151125	-0.00005289	-0.01084473	-0.00405079	0.00859980	-0.05985743	0.92032504	-0.14448094
0.15	0.02977165	0.45436639	-0.00023765	-0.02338827	-0.01219150	0.01684347	-0.11723799	0.65753621	-0.17315477
0.20	0.04837891	0.50561410	-0.00067977	-0.03896204	-0.02528954	0.02530805	-0.17615861	0.37866712	-0.16514593
0.25	0.06722206	0.48563308	-0.00150083	-0.05554063	-0.04241288	0.03224518	-0.22445035	0.13978821	-0.12910533
0.30	0.08362645	0.39426255	-0.00278925	-0.07067895	-0.06161753	0.03617405	-0.25180548	-0.02125162	-0.07846546
0.35	0.09531170	0.24210989	-0.00457157	-0.08188754	-0.08029443	0.03608117	-0.25116909	-0.09185600	-0.02860562
0.40	0.10065770	0.04991577	-0.00679747	-0.08713907	-0.09560949	0.03158399	-0.21987700	-0.08610100	0.00644322
0.45	0.09885222	-0.15417057	-0.00934376	-0.08535635	-0.10498428	0.02301413	-0.16023618	-0.03974271	0.01704815
0.50	0.08996558	-0.33897930	-0.01203614	-0.07671016	-0.10655183	0.01139005	-0.07933402	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.73464769	-0.72980261	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.73537314	-0.73199546	0.00350005	0.01054006	-0.00008904	-0.00295482	-0.00094741	-0.00294366	0.00002399
0.10	0.64540136	-0.63444859	0.00367021	0.01898497	-0.00008932	-0.00308434	-0.00099347	-0.00522885	0.00002404
0.15	0.49707204	-0.47227007	0.00397685	0.02633421	-0.00009056	-0.00332062	-0.00107517	-0.00718647	0.00002434
0.20	0.32674682	-0.28452247	0.00400842	0.02674905	-0.00009089	-0.00334571	-0.00108123	-0.00726070	0.00002439
0.25	0.16954333	-0.11091596	0.00338847	0.02021075	-0.00008225	-0.00284668	-0.00091115	-0.00546697	0.00002199
0.30	0.05281903	0.01499606	0.00212288	0.01101403	-0.00005405	-0.00180125	-0.00056774	-0.00297146	0.00001431
0.35	-0.01020518	0.07485354	0.00064743	0.00400827	-0.00000487	-0.00055443	-0.00016942	-0.00108002	0.00000101
0.40	-0.02414284	0.07247078	-0.00042664	0.00123383	0.00004663	0.00036819	0.00011898	-0.00033489	-0.00001284
0.45	-0.01025132	0.03406174	-0.00066321	0.00107376	0.00006263	0.00057283	0.00018079	-0.00029263	-0.00001705
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.64542753	0.08638555	-0.16466522	0.0	0.0
0.05	0.00079941	-0.68069541	0.06863284	-0.13343048	-0.07685250	0.02085317
0.10	0.00083446	-0.61399770	0.04844962	-0.09528053	-0.14286000	0.03876271
0.15	0.00089742	-0.47965276	0.02944874	-0.05827764	-0.18974787	0.05148355
0.20	0.00090226	-0.31788152	0.01428648	-0.02836352	-0.21069610	0.05716546
0.25	0.00076534	-0.16653860	0.00430445	-0.00853883	-0.20174438	0.05473417
0.30	0.00048167	-0.05363124	-0.00059080	0.00126103	-0.16290760	0.04419407
0.35	0.00014507	0.00798674	-0.00170720	0.00356397	-0.09871167	0.02677362
0.40	-0.00010267	0.02262968	-0.00098634	0.00214799	-0.01791572	0.00484978
0.45	-0.00015614	0.01000032	-0.00017844	0.00046124	0.06764150	-0.01836497
0.50	0.0	0.0	0.0	0.0	0.14488840	-0.03932372

J(CST)= 0.08382106

Table 16

K= 0.25

A1	A2	A3	A4
0.03253375	-0.42006022	-0.00129864	-0.03002557

B1	B2	B3	B4
-0.00889771	0.11447263	0.0	0.00814834

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.43822575	0.02762236
0.05	0.00455175	0.22931051	-0.00000491	-0.00345950	-0.00069039	0.00301593	-0.02090117	1.38382339	-0.09731287
0.10	0.01758504	0.42684764	-0.00006525	-0.01359406	-0.00506362	0.01079128	-0.07478642	1.15327358	-0.18077052
0.15	0.03707013	0.56768578	-0.00029818	-0.02940432	-0.01522774	0.02111108	-0.14630532	0.82156122	-0.21591705
0.20	0.06006404	0.63105476	-0.00086227	-0.04914052	-0.03155769	0.03167098	-0.21948802	0.46918231	-0.20521575
0.25	0.08317089	0.60468966	-0.00191778	-0.07024318	-0.05286139	0.04026253	-0.27902925	0.16753441	-0.15952021
0.30	0.10309029	0.48841166	-0.00358079	-0.08954412	-0.07667655	0.04501852	-0.31198835	-0.03480021	-0.09574711
0.35	0.11709046	0.29593337	-0.00588357	-0.10377401	-0.09971201	0.04467223	-0.30958652	-0.12163854	-0.03331513
0.40	0.12329930	0.05403512	-0.00875460	-0.11027128	-0.11840826	0.03877023	-0.26868105	-0.11140066	0.01008343
0.45	0.12082022	-0.20109951	-0.01202573	-0.10766232	-0.12955326	0.02777809	-0.19249862	-0.05056478	0.02243252
0.50	0.10977751	-0.42975831	-0.01546434	-0.09626323	-0.13086176	0.01303758	-0.09033811	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	0.92044580	-0.89993060	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	0.92015070	-0.90257555	0.00526623	0.01776406	-0.00012631	-0.00455776	-0.00142912	-0.00482555	0.00003430
0.10	0.80693114	-0.78356671	0.00552680	0.03068445	-0.00012672	-0.00475634	-0.00150071	-0.00837572	0.00003438
0.15	0.62047940	-0.58422667	0.00598536	0.04169369	-0.00012854	-0.00511130	-0.00162569	-0.01137626	0.00003484
0.20	0.40577036	-0.35097289	0.00600096	0.04184062	-0.00012877	-0.00512411	-0.00162913	-0.01140396	0.00003486
0.25	0.20713300	-0.13301569	0.00500347	0.03128423	-0.00011453	-0.00430837	-0.00135630	-0.00851664	0.00003093
0.30	0.05990084	0.02601529	0.00302821	0.01685745	-0.00006904	-0.00264038	-0.00081757	-0.00458193	0.00001848
0.35	-0.01844569	0.10092610	0.00077147	0.00610830	0.00000933	-0.00068036	-0.00020369	-0.00165791	-0.00000287
0.40	-0.03376137	0.09565502	-0.00082132	0.00203247	0.00008940	0.00073117	0.00022756	-0.00055426	-0.00002458
0.45	-0.01390516	0.04434650	-0.00109972	0.00186703	0.00010927	0.00098006	0.00030065	-0.00051039	-0.00002984
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.80143172	0.10777283	-0.20306313	0.0	0.0
0.05	0.00123670	-0.84346259	0.08543694	-0.16397524	-0.09607214	0.02618108
0.10	0.00129124	-0.75865316	0.06007765	-0.11643547	-0.17832124	0.04859513
0.15	0.00138798	-0.58999473	0.03620969	-0.07049930	-0.23637283	0.06441498
0.20	0.00139083	-0.38783538	0.01718870	-0.03355508	-0.26165020	0.07130355
0.25	0.00116770	-0.19954783	0.00473894	-0.00929631	-0.24918854	0.06790793
0.30	0.00071277	-0.06010413	-0.00124222	0.00243373	-0.19911504	0.05426285
0.35	0.00017960	0.01467102	-0.00243410	0.00487767	-0.11737680	0.03198884
0.40	-0.00020259	0.03066456	-0.00134770	0.00282163	-0.01536429	0.00418993
0.45	-0.00026793	0.01314223	-0.00023894	0.00058987	0.09165275	-0.02497268
0.50	0.0	0.0	0.0	0.0	0.18700618	-0.05095671

J(CST)= 0.13034236

Table 17

K= 0.30

A1	A2	A3	A4
0.09419775	-0.41984099	-0.00245313	-0.08592981

B1	B2	B3	B4
-0.02582871	0.11628920	0.0	0.02350485

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.71839905	0.03155663
0.05	0.00545977	0.27517158	-0.00000511	-0.00415444	-0.00082604	0.00366468	-0.02498601	1.65873432	-0.11672372
0.10	0.02106553	0.51218331	-0.00007439	-0.01635872	-0.00604237	0.01306526	-0.08907408	1.38519859	-0.21564186
0.15	0.04431167	0.68099636	-0.00035317	-0.03548853	-0.01811854	0.02545511	-0.17353135	0.98720551	-0.25700784
0.20	0.07159013	0.75647604	-0.00104351	-0.05949733	-0.03742754	0.03800135	-0.25903934	0.56191683	-0.24369389
0.25	0.09879798	0.72377872	-0.00235257	-0.08528614	-0.06246228	0.04801178	-0.32724226	0.19669092	-0.18863720
0.30	0.12204230	0.58289230	-0.00443153	-0.10893297	-0.09020883	0.05323804	-0.36281312	-0.04820039	-0.11213392
0.35	0.13821322	0.35094059	-0.00732279	-0.12635106	-0.11669475	0.05219587	-0.35563743	-0.15220356	-0.03756070
0.40	0.14527506	0.06108039	-0.01093399	-0.13422602	-0.13767868	0.04442372	-0.30257571	-0.13776541	0.01381226
0.45	0.14231557	-0.24225461	-0.01504876	-0.13090420	-0.14940232	0.03060100	-0.20826763	-0.06215445	0.02770175
0.50	0.12953836	-0.51085413	-0.01937052	-0.11689359	-0.14928591	0.01247429	-0.08462888	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.10230637	-1.05321217	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.10096931	-1.06146526	0.00654284	0.02494042	-0.00012179	-0.00578987	-0.00181662	-0.00695802	0.00003428
0.10	0.96539068	-0.92653632	0.00691412	0.04313540	-0.00012237	-0.00607381	-0.00191992	-0.01202243	0.00003437
0.15	0.74190623	-0.69455302	0.00755445	0.05843294	-0.00012485	-0.00657236	-0.00209686	-0.01625008	0.00003499
0.20	0.48355842	-0.41818506	0.00755723	0.05828873	-0.00012504	-0.00657564	-0.00209626	-0.01618951	0.00003496
0.25	0.24376953	-0.15604150	0.00614164	0.04318880	-0.00010451	-0.00539994	-0.00170200	-0.01198456	0.00002918
0.30	0.06609815	0.03724410	0.00340237	0.02301028	-0.00003931	-0.00303576	-0.00094061	-0.00637617	0.00001099
0.35	-0.02741589	0.12835091	0.00035717	0.00837898	0.00007137	-0.00031785	-0.00009574	-0.00231701	-0.00001978
0.40	-0.04379572	0.12050390	-0.00166498	0.00317860	0.00017876	0.00153077	0.00046317	-0.00087967	-0.00004953
0.45	-0.01766899	0.05572248	-0.00179337	0.00307868	0.00018888	0.00164950	0.00049676	-0.00085284	-0.00005226
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-0.95090693	0.12871087	-0.23973536	0.0	0.0
0.05	0.00160725	-0.99926221	0.10189438	-0.19309914	-0.11465687	0.03176081
0.10	0.00168624	-0.89698011	0.07146442	-0.13654685	-0.21164525	0.05863278
0.15	0.00182399	-0.69528747	0.04280671	-0.08204627	-0.27868557	0.07721382
0.20	0.00182379	-0.45425034	0.01997262	-0.03837965	-0.30574071	0.08472234
0.25	0.00149633	-0.23044401	0.00508415	-0.00990649	-0.28723097	0.07961106
0.30	0.00083919	-0.06560004	-0.00195531	0.00362522	-0.22385442	0.06207143
0.35	0.00008512	0.02159604	-0.00319370	0.00615855	-0.12344450	0.03427086
0.40	-0.00042583	0.03862543	-0.00171896	0.00346103	-0.00039862	0.00019576
0.45	-0.00045690	0.01621985	-0.00029935	0.00070186	0.12639964	-0.03492430
0.50	0.0	0.0	0.0	0.0	0.23677182	-0.06550014

J(CST)= 0.18580180

Table 18

K= 0.35

A1	A2	A3	A4
0.11861956	-0.41593575	-0.00272688	-0.10866952

B1	B2	B3	B4
-0.03261961	0.11653215	0.0	0.02983586

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.00811005	0.03531830
0.05	0.00636703	0.32103455	-0.00000592	-0.00485040	-0.00095378	0.00427879	-0.02883946	1.94042110	-0.13606215
0.10	0.02453420	0.59753746	-0.00008780	-0.01913894	-0.00696899	0.01523224	-0.10265273	1.62167549	-0.25027400
0.15	0.05149725	0.79434210	-0.00042101	-0.04164172	-0.02087151	0.02962671	-0.19962859	1.15554905	-0.29781014
0.20	0.08295733	0.88186574	-0.00125203	-0.07003236	-0.04305419	0.04413563	-0.29733706	0.65575826	-0.28192472
0.25	0.11409342	0.84256876	-0.00283456	-0.10066003	-0.07173324	0.05560485	-0.37451851	0.22572756	-0.21757942
0.30	0.14044243	0.67653483	-0.00535246	-0.12880349	-0.10338759	0.06141179	-0.41350400	-0.06233459	-0.12840456
0.35	0.15857059	0.40433246	-0.00885330	-0.14949888	-0.13340229	0.05984773	-0.40279067	-0.18341911	-0.04172569
0.40	0.16633874	0.06561810	-0.01321700	-0.15873158	-0.15688062	0.05042483	-0.33910906	-0.16427094	0.01759459
0.45	0.16284722	-0.28677613	-0.01817198	-0.15455210	-0.16952580	0.03400966	-0.22831953	-0.07348734	0.03299528
0.50	0.14835578	-0.59602714	-0.02335254	-0.13768178	-0.16844964	0.01272193	-0.08472705	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.28633881	-1.20622540	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.28405666	-1.21777725	0.00821050	0.03349557	-0.00012949	-0.00743585	-0.00231830	-0.00941953	0.00003774
0.10	1.12633610	-1.06687164	0.00870640	0.05785942	-0.00013021	-0.00781547	-0.00245814	-0.01629415	0.00003787
0.15	0.86578119	-0.80327040	0.00955437	0.07818121	-0.00013347	-0.00847858	-0.00269647	-0.02200754	0.00003871
0.20	0.56321442	-0.48460537	0.00953759	0.07769775	-0.00013351	-0.00846629	-0.00269177	-0.02187308	0.00003864
0.25	0.28127891	-0.17832685	0.00763066	0.05725897	-0.00010482	-0.00685733	-0.00215494	-0.01612014	0.00003049
0.30	0.07220608	0.04958965	0.00399819	0.03030419	-0.00001456	-0.00365295	-0.00113113	-0.00852366	0.00000498
0.35	-0.03697763	0.15700454	0.00002722	0.01107663	0.00013696	-0.00001105	-0.00001228	-0.00310662	-0.00003779
0.40	-0.05436444	0.14612514	-0.00251878	0.00451287	0.00027950	0.00238961	0.00070361	-0.00126122	-0.00007794
0.45	-0.02162218	0.06722283	-0.00252356	0.00446472	0.00028055	0.00239462	0.00070515	-0.00124768	-0.00007832
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.09941769	0.14995527	-0.27612215	0.0	0.0
0.05	0.00209983	-1.15392494	0.11862046	-0.22198051	-0.13224721	0.03705844
0.10	0.00220686	-1.03425026	0.08306175	-0.15647584	-0.24354684	0.06826091
0.15	0.00239322	-0.79974955	0.04953339	-0.09346908	-0.31976026	0.08964407
0.20	0.00238976	-0.52008808	0.02279951	-0.04312513	-0.34934527	0.09797013
0.25	0.00193681	-0.26096851	0.00540666	-0.01046845	-0.32597429	0.09146130
0.30	0.00103364	-0.07086706	-0.00271826	0.00484516	-0.25072414	0.07041383
0.35	0.00000751	0.02865056	-0.00399532	0.00745547	-0.13309652	0.03748479
0.40	-0.00066752	0.04664376	-0.00210973	0.00410904	0.00972344	-0.00251518
0.45	-0.00066911	0.01931041	-0.00036325	0.00081805	0.15544188	-0.04334184
0.50	0.0	0.0	0.0	0.0	0.28053224	-0.07840341

J(CST)= 0.25103605

Table 19

K= 0.40

A1	A2	A3	A4
0.17233133	-0.41614527	-0.00272688	-0.16009021

B1	B2	B3	B4
-0.04807312	0.11814445	0.0	0.04464457

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.28961658	0.03840600
0.05	0.00727349	0.36689472	-0.00000587	-0.00554737	-0.00108821	0.00494323	-0.03287833	2.21809959	-0.15571618
0.10	0.02798994	0.68284833	-0.00009598	-0.02193383	-0.00793198	0.01753946	-0.11663651	1.85638523	-0.28489989
0.15	0.05862005	0.90748399	-0.00047720	-0.04785797	-0.02369249	0.03398454	-0.22594768	1.32233143	-0.33827776
0.20	0.09414333	1.00669575	-0.00144631	-0.08072436	-0.04872685	0.05039349	-0.33495939	0.74679720	-0.31947511
0.25	0.12900937	0.96027780	-0.00331171	-0.11631352	-0.08090180	0.06311053	-0.41935611	0.25038058	-0.24552137
0.30	0.15821499	0.76863718	-0.00629752	-0.14906394	-0.11611742	0.06913143	-0.45917028	-0.08165985	-0.14347881
0.35	0.17807567	0.45620358	-0.01046117	-0.17309743	-0.14906645	0.06655347	-0.44176567	-0.21932918	-0.04472621
0.40	0.18643779	0.06961185	-0.01565592	-0.18369228	-0.17418575	0.05492758	-0.36418277	-0.19394082	0.02225012
0.45	0.18247932	-0.32955253	-0.02155279	-0.17863786	-0.18667936	0.03539661	-0.23405463	-0.08619010	0.03870462
0.50	0.16653943	-0.67576689	-0.02771266	-0.15893245	-0.18344074	0.01055000	-0.06863111	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.46862888	-1.34616947	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.46485043	-1.36512947	0.00921332	0.04312031	-0.00006360	-0.00849326	-0.00265915	-0.01238253	0.00002085
0.10	1.28467178	-1.20201111	0.00984931	0.07428998	-0.00006437	-0.00898120	-0.00284060	-0.02128132	0.00002099
0.15	0.98667753	-0.90930146	0.01091914	0.09992784	-0.00006825	-0.00982207	-0.00314525	-0.02858451	0.00002202
0.20	0.63938105	-0.54898524	0.01085824	0.09867668	-0.00006789	-0.00977429	-0.00312805	-0.02823200	0.00002184
0.25	0.31490618	-0.19770133	0.00839159	0.07207447	-0.00003000	-0.00766047	-0.00242303	-0.02062984	0.00001093
0.30	0.07457864	0.06603444	0.00380458	0.03776129	0.00008792	-0.00352534	-0.00110885	-0.01080015	-0.00002293
0.35	-0.04940974	0.19004035	-0.00106993	0.01395473	0.00028235	0.00106746	0.00028862	-0.00397585	-0.00007874
0.40	-0.06650680	0.17510802	-0.00398590	0.00639869	0.00045440	0.00390266	0.00112443	-0.00181051	-0.00012812
0.45	-0.02601682	0.08032805	-0.00359209	0.00649770	0.00042250	0.00351561	0.00101572	-0.00183749	-0.00011940
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.24497509	0.17071396	-0.31112915	0.0	0.0
0.05	0.00245222	-1.30471420	0.13483840	-0.24948788	-0.15053982	0.04274925
0.10	0.00259141	-1.16704369	0.09414679	-0.17512721	-0.27582878	0.07834989
0.15	0.00283084	-0.89946854	0.05576229	-0.10378963	-0.35988009	0.10225946
0.20	0.00281734	-0.58129895	0.02517296	-0.04700355	-0.38978064	0.11080575
0.25	0.00221315	-0.28740841	0.00535972	-0.01042686	-0.35875297	0.10205674
0.30	0.00102843	-0.07297647	-0.00373490	0.00641796	-0.26871830	0.07654935
0.35	-0.00028828	0.03772524	-0.00493158	0.00890819	-0.13141912	0.03760941
0.40	-0.00110093	0.05561725	-0.00254655	0.00479437	0.03261046	-0.00893925
0.45	-0.00099409	0.02264258	-0.00043220	0.00092922	0.19724548	-0.05568236
0.50	0.0	0.0	0.0	0.0	0.33543611	-0.09493959

J(CST)= 0.32519448

Table 20

K= 0.45

A1	A2	A3	A4
0.18263614	-0.41227615	-0.00258923	-0.17017704

B1	B2	B3	B4
-0.05107837	0.11765712	0.0	0.04759321

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.58677292	0.04157488
0.05	0.00817919	0.41275775	-0.00000706	-0.00624536	-0.00121226	0.00553469	-0.03661974	2.50638485	-0.17522949
0.10	0.03143390	0.76818842	-0.00011278	-0.02474432	-0.00883135	0.01962396	-0.12981027	2.09790993	-0.31935030
0.15	0.06568658	1.02069473	-0.00055778	-0.05414299	-0.02636258	0.03799028	-0.25123638	1.49327850	-0.37862086
0.20	0.10516793	1.13154507	-0.00168818	-0.09159172	-0.05417811	0.05626700	-0.37198913	0.84016800	-0.35701883
0.25	0.14358610	1.07771206	-0.00386305	-0.13228542	-0.08986825	0.07034582	-0.46488774	0.27619660	-0.27357864
0.30	0.17542011	0.85978341	-0.00733955	-0.16977316	-0.12882918	0.07685834	-0.50766069	-0.10032219	-0.15873855
0.35	0.19679123	0.50599658	-0.01217533	-0.19719964	-0.16511965	0.07368982	-0.48634708	-0.25450313	-0.04791192
0.40	0.20558733	0.06996769	-0.01818655	-0.20908761	-0.19253600	0.06038190	-0.39795268	-0.22259688	0.02677849
0.45	0.20107692	-0.37792885	-0.02497802	-0.20294273	-0.20576727	0.03828245	-0.25143552	-0.09794849	0.04436665
0.50	0.18362743	-0.76339030	-0.03203227	-0.18004823	-0.20142901	0.01036597	-0.06651324	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.65412617	-1.49066448	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.64910793	-1.51237679	0.01106663	0.05496303	-0.00004569	-0.01042157	-0.00322049	-0.01561236	0.00001766
0.10	1.44670582	-1.33551788	0.01185620	0.09384453	-0.00004662	-0.01102753	-0.00344874	-0.02686111	0.00001785
0.15	1.11114979	-1.01407051	0.01317380	0.12555671	-0.00005154	-0.01206738	-0.00383116	-0.03607018	0.00001921
0.20	0.71840572	-0.61289352	0.01306012	0.12346518	-0.00005068	-0.01197564	-0.00380490	-0.03557099	0.00001894
0.25	0.35022914	-0.21698374	0.00995049	0.08976960	-0.00000073	-0.00926855	-0.00291349	-0.02591322	0.00000455
0.30	0.07750499	0.08254385	0.00425209	0.04682099	0.00015282	-0.00401883	-0.00126808	-0.01351323	-0.00003986
0.35	-0.06198820	0.22302020	-0.00171989	0.01741988	0.00040319	0.00176157	0.00046076	-0.00500290	-0.00011240
0.40	-0.07894737	0.20367992	-0.00519413	0.00839864	0.00061901	0.00524430	0.00146849	-0.00238689	-0.00017507
0.45	-0.03053224	0.09283346	-0.00455528	0.00856864	0.00056385	0.00459613	0.00129333	-0.00243301	-0.00016002
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.39044952	0.19195843	-0.34622544	0.0	0.0
0.05	0.00303697	-1.45553207	0.15149122	-0.27711409	-0.16761333	0.04784887
0.10	0.00321211	-1.30006313	0.10558665	-0.19391698	-0.30675346	0.08759952
0.15	0.00351389	-0.99963272	0.06223867	-0.11424828	-0.39959592	0.11415988
0.20	0.00349279	-0.64309591	0.02767975	-0.05099716	-0.43171024	0.12340331
0.25	0.00271676	-0.31443781	0.00535388	-0.01046288	-0.39556897	0.11317039
0.30	0.00120091	-0.07555544	-0.00476174	0.00795289	-0.29351956	0.08411938
0.35	-0.00047246	0.04650915	-0.00589015	0.01035363	-0.13907862	0.04009697
0.40	-0.00148265	0.06446332	-0.00299668	0.00548802	0.04433066	-0.01222002
0.45	-0.00130493	0.02594389	-0.00050434	0.00104967	0.22713578	-0.06439537
0.50	0.0	0.0	0.0	0.0	0.37898755	-0.10776675

J(CST)= 0.40921038

Table 21

K= 0.50

A1	A2	A3	A4
0.19594330	-0.40720999	-0.00231049	-0.18355191

B1	B2	B3	B4
-0.05502446	0.11742461	0.0	0.05157545

TIME(SEC)	Y1	Y2	Y3	Y4	Y5	Y6	Y7	P1	P2
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.88441467	0.04469473
0.05	0.00908413	0.45862234	-0.00000815	-0.00694438	-0.00132953	0.00613254	-0.04015275	2.79641151	-0.19416565
0.10	0.03486575	0.85354280	-0.00012921	-0.02757015	-0.00967863	0.02172380	-0.14219058	2.34234333	-0.35287726
0.15	0.07269502	1.13393974	-0.00063901	-0.06049514	-0.02886847	0.04200978	-0.27486926	1.66762638	-0.41798002
0.20	0.11602646	1.25643253	-0.00193593	-0.10263026	-0.05927208	0.06213282	-0.40635699	0.93651140	-0.39373827
0.25	0.15781862	1.19515419	-0.00443295	-0.14857149	-0.09820461	0.07752734	-0.50676274	0.30392605	-0.30109042
0.30	0.19206691	0.95096284	-0.00842226	-0.19094121	-0.14057541	0.08446157	-0.55167872	-0.11816543	-0.17376226
0.35	0.21477199	0.55606872	-0.01396144	-0.22187042	-0.17984033	0.08061898	-0.52598447	-0.28953761	-0.05113266
0.40	0.22394407	0.07120353	-0.02082734	-0.23511028	-0.20919889	0.06554753	-0.42678189	-0.25132811	0.03110377
0.45	0.21898472	-0.42445856	-0.02855543	-0.22789079	-0.22287387	0.04081764	-0.26441407	-0.10975647	0.04982385
0.50	0.20027351	-0.84794921	-0.03654683	-0.20180136	-0.21724766	0.00980376	-0.06103128	0.0	0.0

TIME(SEC)	P3	P4	P5	P6	P7	P8	P9	P10	P11
0.0	1.83884811	-1.62665272	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	1.83288574	-1.65229797	0.01270534	0.06631547	-0.00000204	-0.01221316	-0.00376827	-0.01902638	0.00000839
0.10	1.60900974	-1.46441078	0.01366458	0.11371279	-0.00000311	-0.01294984	-0.00404898	-0.03290838	0.00000864
0.15	1.23655415	-1.11718178	0.01525904	0.15222609	-0.00000918	-0.01421353	-0.00451881	-0.04426350	0.00001036
0.20	0.79855090	-0.67708176	0.01510319	0.14944839	-0.00000787	-0.01408587	-0.00448402	-0.04362288	0.00001001
0.25	0.38639998	-0.23701316	0.01132690	0.10830277	0.00005524	-0.01074661	-0.00338943	-0.03169826	-0.00000835
0.30	0.08084893	0.09881765	0.00448070	0.05626784	0.00024786	-0.00430147	-0.00138575	-0.01647106	-0.00006479
0.35	-0.07445568	0.25638610	-0.00259885	0.02107775	0.00055893	0.00273772	0.00069471	-0.00613132	-0.00015626
0.40	-0.09139305	0.23283237	-0.00659245	0.01064649	0.00081958	0.00686519	0.00187432	-0.00305121	-0.00023330
0.45	-0.03506069	0.10570222	-0.00562565	0.01090696	0.00073126	0.00585241	0.00161001	-0.00312176	-0.00020923
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TIME(SEC)	P12	P13	P14	P15	V1	V2
0.0	0.0	-1.53254986	0.21323186	-0.38060665	0.0	0.0
0.05	0.00363064	-1.60295868	0.16822016	-0.30422193	-0.18370086	0.05299553
0.10	0.00384618	-1.43029022	0.11713254	-0.21240544	-0.33567852	0.09688640
0.15	0.00421850	-1.09791660	0.06881768	-0.12459320	-0.43640190	0.12603235
0.20	0.00419013	-0.70395625	0.03025968	-0.05500691	-0.47005475	0.13585848
0.25	0.00322221	-0.34129089	0.00538565	-0.01057992	-0.42847431	0.12399393
0.30	0.00133591	-0.07842636	-0.00577597	0.00939703	-0.31454015	0.09125012
0.35	-0.00073267	0.05484825	-0.00684692	0.01173307	-0.14359653	0.04203534
0.40	-0.00195179	0.07296979	-0.00344707	0.00615024	0.05808861	-0.01608727
0.45	-0.00167492	0.02912753	-0.00057642	0.00116251	0.25764799	-0.07364494
0.50	0.0	0.0	0.0	0.0	0.42166096	-0.12099797

J(CST)= 0.50186658

Table 22

```
 REALISTIC FEEDBACK CONTROL OF TURBOGENERATORS;
 THE ALTERNATER IS CONTROLLED THROUGH A LINEAR FEEDBACK OF
 THE STATE VARIABLES.
 THE FEEDBACK PARAMETERS ARE OBTAINED BY SOLVING A TWO POINT
 NONLINEAR BOUNDARY VALUE PROBLEM.
 THE VALUES TO BE OBTAINED FOR THESE PARAMETERS DEPEND ON
 THE STRENGTH AND DURATION OF THE DISTURBANCE SINCE THE MODEL IS
 NONLINEAR CONTRARY TO THE USUAL FEEDBACK CONTROL OF A LINEAR MODEL

DIMENSION TBL(41,25),Y(88),DY(88),A(4),B(4),AA(4),BB(4),V(2),
*         JCOST(31),PLOT(21,8)
REAL K,KK,JCOST,JOPT,K1,K2
INTEGER SW
DATA A,B,AA,BB/16*0.0/,JCOST/31*0.0/
 DATA A/0.01633753,-0.41949201,-0.00091682,-0.01575423/,
*     B/-0.00453819,0.11382747,0.0,0.00430573/
K=-0.05
DO 99 II=1,11
K=K+0.05

SW=1
CALL PARAMT(A,B,K)
 'PARAMT' IS THE SUBROUTINE WHICH OBTAINS THE OPTIMAL FEEDBACK
 PARAMETERS BY SOLVING THE TWO POINT NONLINEAR BOUNDARY VALUE PROBLEM.

DO 10 J=1,88
Y(J)=0.0
0   DY(J)=0.0
 INITIAL Y,DY WITH ZERO.

DO 15 J=1,25
DO 15 I=1,41
5   TBL(I,J)=0.0
 INITIAL TBL WITH ZERO.

N=22
T=0.0
TF=0.5
L=0
M=1
JJ=0
 N IS NO. OF EQ.
 T IS INITIAL TIME.
 TF IS FINAL TIME.
 L IS A PARAMETER IN "RUNTA".
 M IS NO. OF INTERVAL.
 JJ IS COUNTER.

0   CALL OPTIM(Y,DY,A,B,SW,K)
 SUBROUTINE OF OPTIMAL EQUATIONS
CALL RUNTA(7,L,I,Y,DY,T,0.005)
 SUBROUTINE OF RUNGE-KUTTA METHOD
GO TO (20,21),I
1   JJ=JJ+1
IF(MOD(JJ,10)) 20,22,20
2   M=M+1
CALL VVKK(Y,A,B,V,K1,K2)
 SOLVE CONTROLS V1,V2.
DO 31 J=1,7
```

```
   TBL(M,J)=Y(J)
   DO 32 J=23,24
2  TBL(M,J)=V(J-22)
   JCOST(M)=COST(Y,A,B,V)
   IF(M-11) 20,64,64

50  THE NON-LINEAR STATE EQUATIONS(Y1---Y7) ON THE FEEDBACK ARE INTEGRATED
    BY USING RUNGE-KUTTA METHOD
    FORWARDS FROM T=0.0 SEC TO TF=0.5 SEC BETWEEN 20 LINE AND 50 LINE.

4  CALL SIMP(JCOST,JOPT)
    J(OPT) IS OBTAINED BY USING THE SIMPSON'S METHOD.

    CALL EIGENV(Y(8),Y(9),Y(11),EI,K1,K2)
    FIND THE SMALLEST EIGENVALUE OF 'L'.

   TBL(11,25)=EI
   JJ=0
   DO 65 J=1,88
   Y(J)=0.0
5  DY(J)=0.0

   SW=2
   N=22
   T=0.5
   TF=0.0
   L=0
   M=11
    IF SW=1,IT IS THE FIRST STEP FOR Y1---Y7.
    IF SW=2,IT IS THE SECOND STEP FOR P1---P15.
    IF SW=3,IT IS THE LAST STEP FROM T=0.5 SEC TO TF=2.0 SEC.
    N IS NO. OF EQ.
    T IS THE INITIAL TIME.
    TF IS THE FINAL TIME.
    L IS A PARAMETER IN "RUNTA".
    M IS NO. OF INTERVAL.

6  M=M-1
   DO 68 J=1,7
   DY(J)=0.0
8  Y(J)=TBL(M,J)
0  CALL OPTIM(Y,DY,A,B,SW,K)
   CALL RUNTA(22,L,I,Y,DY,T,-0.005)
   GO TO (70,71),I
1  JJ=JJ+1
   IF(MOD(JJ,10)) 70,72,70
2  JJ=0
   DO 81 J=8,22
1  TBL(M,J)=Y(J)
   IF(M-1) 85,85,66


    THE NON-LINEAR COSTATE EQUATIONS(P1----P15) ARE INTEGRATED BY
    USING THE SAME METHOD BACKWARDS FROM T=0.5 SEC TO TF=0.0 SEC
    BETWEEN 66 LINE AND 81 LINE.

5  DO 86 J=12,19
6  TBL(1,J)=0.0

   SW=3
```

```
KK=0.0
IF(SW.EQ.3) GO TO 94

DO 87 I=1,22
Y(I)=0.0
DY(I)=0.0

DO 89 I=1,4
Y(I)=TBL(11,I)
AA(I)=0.0
BB(I)=0.0

T=0.5
TF=2.0
M=0
J=11
JJ=0
L=0
CALL OPTIM(Y,DY,AA,BB,SW,KK)
CALL RUNTA(22,L,I,Y,DY,T,0.005)
GO TO (90,91),I
JJ=JJ+1
IF(MOD(JJ,10)) 90,911,90

1   J=J+1
DO 92 I=1,22
TBL(J,I)=Y(I)

CALL VVKK(Y,AA,BB,V,K1,K2)
DO 93 I=23,24
TBL(J,I)=V(I-22)
IF(J-41) 90,94,94

 T IS THE INITIAL TIME TO START AFTER THE DISTURBANCE.
 TF IS THE FINAL TIME.
 M IS NO. OF INITIAL.
 J IS NO. OF TBL-ROW.
 FIND OUT WHAT THE DISTURBANCE IS AFTER 0.5 SEC
 BETWEEN 85 LINE AND 93 LINE.

CALL WRITER(K,A,B,JOPT,TBL)
CONTINUE
STOP
END

 NUMERICAL SOLUTIONS FOR OPTIMAL PARAMETERS
 USING GRADIENT TECHNIQUES;

SUBROUTINE PARAMT(A,B,K)
DIMENSION Y(88),DY(88),A(4),B(4),AK(4),BK(4),G(4),H(4),GG(4),
*          HH(4),TBL(11,22),JCOST(31),JOPT(31),JJ(31),V(2)
INTEGER SW
REAL JCOST,K,JOPT,JJ,K1,K2,JK1,JK,JD
DATA X/1.0/,TBL/242*0.0/,JCOST,JOPT/62*0.0/,EPSI/0.0001/
*     ,JJ/31*0.0/,JK/0.0/

 Y IS THE STATE FUNCTIONS
 DY IS THE DIFFENTIAL FUNCTIONS
 A IS THE FEEDBACK PARAMETERS A(I) A1,A2,A3,A4
 B IS THE FEEDBACK PARAMETERS B(I) B1,B2,B3,B4
```

```
 AK IS A(I+1)
 BK IS B(I+1)
 G IS DH/DA
 H IS DH/DB
 GG IS (DH/DA)**2
 HH IS (DH/DB)**2
 JK IS THE COST FUNCTIONAL J(I)
 JK1 IS THE COST FUNCTIONAL J(I+1)
 JD IS J(DELDA)=J(I)-J(I+1)
 K IS THE STRENGTH OF TORQUE PULSE
 EPSI IS THE EPSILON
 X IS THE ALPHA

DO 25 II=1,20

DO 2 I=1,88
Y(I)=0.0
DY(I)=0.0

T=0.0
M=0
J=1
SW=1
JJJ=0
L=0
CALL OPTIM(Y,DY,A,B,SW,K)
CALL RUNTA(7,L,I,Y,DY,T,0.005)
GO TO (5,6),I
JJJ=JJJ+1
IF(MOD(JJJ,10)) 5,66,5

 J=J+1
DO 7 I=1,7
TBL(J,I)=Y(I)
CALL VVKK(Y,A,B,V,K1,K2)
JJ(J)=COST(Y,A,B,V)
IF(J-11) 5,88,88
CALL SIMP(JJ,AREA)
JK1=AREA

J=J-1
DO 9 I=1,7
Y(I)=TBL(J,I)
DY(I)=0.0

T=0.5
M=0
SW=2
JJJ=0
L=0
CALL OPTIM(Y,DY,A,B,SW,K)
CALL RUNTA(22,L,I,Y,DY,T,-0.005)
GO TO (10,11),I
JJJ=JJJ+1
IF(MOD(JJJ,10)) 10,111,10

 DO 13 I=12,19
TBL(J,I)=DY(I)
IF(J-2) 14,14,8
```

```
DO 16 I=12,15
DO 15 J=1,11
JCOST(J)=TBL(J,I)
JOPT(J)=TBL(J,I)**2
CALL SIMP(JCOST,AREA)
G(I-11)=AREA/0.5
CALL SIMP(JOPT,AREA)
GG(I-11)=AREA

DO 18 I=16,19
DO 17 J=1,11
JCOST(J)=TBL(J,I)
JOPT(J)=TBL(J,I)**2
CALL SIMP(JCOST,AREA)
H(I-15)=AREA/0.5
CALL SIMP(JOPT,AREA)
HH(I-15)=AREA

IF(K.GT.0.4) X=0.1
IF(K.GT.1.3) X=0.01
IF(K.GT.2.0) X=0.01

DO 20 I=1,4
AK(I)=A(I)-X*G(I)
BK(I)=B(I)-X*H(I)
JD=JK-JK1

DO 21 I=1,4
IF(ABS(G(I)).GT.EPSI.AND.ABS(JD).GT.EPSI) A(I)=AK(I)
IF(ABS(H(I)).GT.EPSI.AND.ABS(JD).GT.EPSI) B(I)=BK(I)

JK=JK1
IF(ABS(JD).LE.EPSI) GO TO 27
CONTINUE

IF(II.GE.1) GO TO 27
DO 27 I=1,4
IF(SQRT(GG(I)).GT.EPSI) GO TO 1
IF(SQRT(HH(I)).GT.EPSI) GO TO 1
CONTINUE

 PRINT 30,II
FORMAT(I5)
RETURN
END

 OPTIMAL EQUATIONS Y1---Y7,P1---P15

 SUBROUTINE OPTIM(Y,DY,A,B,SW,K)
 DIMENSION Y(88),DY(88),A(4),B(4),V(2)
 INTEGER SW
 REAL M,KD,K,K1,K2,N1,N2,N3,N4

 DATA M/0.04225/,KD/0.02535/,S4/-1.76/,S5/1.46/,AA/0.812/,C/0.98/,
*     X3S/1.874/,U1S/0.8/,A1/2.5/,A2/1.0/,A3/0.1/,A4/2.5/,B1/1.0/,
*     B2/1.0/,X1S/0.603/,X4S/0.798/,C1/0.01/,C2/0.01/,C3/0.01/,
*    G1/0.00188/,G2/1.33/,G3/1.42/,TG/0.2/,TB/0.49/,TE/0.2/,CR/1.0/
 DATA N1,N2,N3,N4,Q1,Q2,Q3,Q4/8*0.001/

 CALL VVKK(Y,A,B,V,K1,K2)
```

```
  Y1,Y2,Y3,Y4,Y5,Y6,Y7 OPTIMAL EQUATIONS.
 IF(SW.EQ.2) GO TO 10

 DY(1)=X4S*Y(2)+Y(2)*Y(4)
 DY(2)=1/M*(Y(5)-S4*X4S*Y(1)-S5*X3S*Y(1)-KD*Y(2)-S5*X1S*Y(3)
*       -S4*X1S*Y(4)-S4*Y(1)*Y(4)-S5*Y(1)*Y(3)+K*U1S)
 DY(3)=Y(6)-AA*Y(3)+C*Y(4)
 DY(4)=-Y(1)*Y(2)-Y(2)*X1S
 DY(5)=1/TB*(Y(7)-Y(5))
 DY(6)=1/TE*(CR*V(2)-Y(6))
 DY(7)=1/TG*(G2*G3*V(1)-Y(7))

  P1,P2,-------,P14,P15 COSTATE OPTIMAL EQUATIONS
 IF(SW.EQ.1) GO TO 20

 DY(8)=2.0*A1*Y(1)+Y(11)*Y(2)+Y(9)*S5*Y(3)/M+Y(9)*S4*Y(4)/M
*      -K1*A(1)-K2*B(1)+Y(9)*S4*X4S/M+Y(9)*S5*X3S/M
 DY(9)=2.0*A2*Y(2)+Y(11)*Y(1)-Y(8)*Y(4)-K1*A(2)-K2*B(2)+X1S*Y(11)
*      -X4S*Y(8)+KD*Y(9)/M
 DY(10)=2.0*A3*Y(3)+Y(9)*S5*Y(1)/M-K1*A(3)-K2*B(3)+AA*Y(10)+
*        S5*X1S*Y(9)/M
 DY(11)=2.0*A4*Y(4)-Y(8)*Y(2)+Y(9)*S4*Y(1)/M-K1*A(4)-K2*B(4)
*       -C*Y(10)+S4*X1S*Y(9)/M
 DY(12)=2.0*N1*A(1)-K1*Y(1)
 DY(13)=2.0*N2*A(2)-K1*Y(2)
 DY(14)=2.0*N3*A(3)-K1*Y(3)
 DY(15)=2.0*N4*A(4)-K1*Y(4)
 DY(16)=2.0*Q1*B(1)-K2*Y(1)
 DY(17)=2.0*Q2*B(2)-K2*Y(2)
 DY(18)=2.0*Q3*B(3)-K2*Y(3)
 DY(19)=2.0*Q4*B(4)-K2*Y(4)
 DY(20)=2.0*C1*Y(5)+Y(20)/TB-Y(9)/M
 DY(21)=2.0*C2*Y(6)+Y(21)/TE-Y(10)
 DY(22)=2.0*C3*Y(7)+Y(22)/TG-Y(20)/TB
 CONTINUE
 RETURN
 END

  CHECK POSITIVE DEFINITE OF 'L'.

 SUBROUTINE EIGENV(P1,P2,P4,EIGEN,K1,K2)
 DIMENSION A(17,17),WK(17)
 COMPLEX W(17),Z(17,17),ZN
 REAL P1,P2,P4,K1,K2,S4,S5,M,N1,N2,N3,N4
 INTEGER N,IA,IJOB,IZ,IER
 DATA A1/2.5/,A2/1.0/,A3/0.1/,A4/2.5/,B1/1.0/,B2/1.0/,S4/-1.76/,
*      S5/1.46/,M/0.04225/,C1,C2,C3/3*0.01/
 DATA N1,N2,N3,N4,Q1,Q2,Q3,Q4/8*0.001/

 IA=17
 IZ=17
 N=17
 IJOB=0
 DO 10 J=1,17
 DO 10 K=1,17
 A(J,K)=0.0

 A(1,1)=A1
 A(1,2)=P4/2.0
```

```
A(1,3)=(P2*S5)/(2.0*M)
A(1,4)=(P2*S4)/(2.0*M)
A(1,5)=-K1/2.0
A(1,9)=-K2/2.0
A(2,1)=P4/2.0
A(2,2)=A2
A(2,4)=-P1/2.0
A(2,6)=-K1/2.0
A(2,10)=-K2/2.0
A(3,1)=(P2*S5)/(2.0*M)
A(3,3)=A3
A(3,7)=-K1/2.0
A(3,11)=-K2/2.0
A(4,1)=(P2*S4)/(2.0*M)
A(4,2)=-P1/2.0
A(4,4)=A4
A(4,8)=-K1/2.0
A(4,12)=-K2/2.0
A(5,1)=-K1/2.0
A(5,5)=N1
A(6,2)=-K1/2.0
A(6,6)=N2
A(7,3)=-K1/2.0
A(7,7)=N3
A(8,4)=-K1/2.0
A(8,8)=N4
A(9,1)=-K2/2.0
A(9,9)=Q1
A(10,2)=-K2/2.0
A(10,10)=Q2
A(11,3)=-K2/2.0
A(11,11)=Q3
A(12,4)=-K2/2.0
A(12,12)=Q4
A(13,13)=B1
A(14,14)=B2
A(15,15)=C1
A(16,16)=C2
A(17,17)=C3

CALL EIGRF(A,N,IA,IJOB,W,Z,IZ,WK,IER)
 WRITE(6,20)
FORMAT('0',12X,'REAL',10X,'IMAG' /)
 WRITE(6,30) W
FORMAT(2X,2F16.4)
DO 40 I=1,17
WK(I)=REAL(W(I))
EIGEN=AMIN1(WK(1),WK(2),WK(3),WK(4),WK(5),WK(6),WK(7),WK(8),WK(9),
*   WK(10),WK(11),WK(12),WK(13),WK(14),WK(15),WK(16),WK(17))
RETURN
END

 PRINT THE OUTPUT.

SUBROUTINE WRITER(K,A,B,JOPT,TBL)
DIMENSION A(4),B(4),TBL(41,25)
REAL K,JOPT

WRITE(6,10) K
FORMAT(/////  ' K= ',F4.2 /)
```

```
 WRITE(6,15)
 FORMAT( '    A1              A2                A3                A4    ' )
 WRITE(6,16) (A(I),I=1,4)
 FORMAT(1H ,4F12.8 /)

 WRITE(6,17)
 FORMAT( '    B1              B2                B3                B4    ' )
 WRITE(6,18) (B(I),I=1,4)
 FORMAT(1H ,4F12.8 )

 WRITE(6,19)
 FORMAT(//'    TIME(SEC)    Y1                Y2                Y3                Y4
*          Y5                Y6                Y7                P1                P2 '/)

 T=0.0
 DO 21 I=1,11
 WRITE(6,20) (T,(TBL(I,J),J=1,9))
 FORMAT(1H ,F6.2,6X,9F12.8)
 T=T+0.05

 WRITE(6,23)
 FORMAT( /'    TIME(SEC)    P3                P4                P5                P6
*          P7                P8                P9                P10               P11'/)

 T=0.0
 DO 25 I=1,11
 WRITE(6,24) (T,(TBL(I,J),J=10,18))
 FORMAT(1H ,F6.2,6X,9F12.8)
 T=T+0.05

 WRITE(6,37)
 FORMAT( /'    TIME(SEC)    P12               P13               P14               P15
*          V1                V2                          EIGENV ' /)

 T=0.0
 DO 28 I=1,11
 WRITE(6,35) (T,(TBL(I,J),J=19,24))
 FORMAT(1H ,F6.2,6X,6F12.8,F12.4)
 T=T+0.05

 WRITE(6,30) JOPT
 FORMAT( // ' J(CST)=',F12.8 // )

 IF(I.GT.1) GO TO 38
 WRITE(6,19)
 T=0.5
 DO 32 I=11,41
 WRITE(6,20) (T,(TBL(I,J),J=1,9))
 T=T+0.05

 WRITE(6,23)
 T=0.5
 DO 34 I=11,41
 WRITE(6,24) (T,(TBL(I,J),J=10,18))
 T=T+0.05

 WRITE(6,37)
 T=0.5
 DO 36 I=11,41
```

```
      WRITE(6,35) (T,(TBL(I,J),J=19,24))
5     FORMAT(1H ,F6.2,6X,6F12.8)
6     T=T+0.05
7     FORMAT( /'   TIME(SEC)   P12          P13          P14          P15
     *       V1           V2 '/)

8     RETURN
      END

       RUNGE-KUTTA METHOD.

      SUBROUTINE RUNTA(N,K,I,X,DX,T,H)
      DIMENSION Y(100),Z(100),X(88),DX(88)

      K=K+1
      GO TO (1,2,3,4,5),K

      DO 10 J=1,N
      Z(J)=DX(J)
      Y(J)=X(J)
0     X(J)=Y(J)+0.5*H*DX(J)
5     T=T+0.5*H
      I=1
      RETURN

      DO 15 J=1,N
      Z(J)=Z(J)+2.0*DX(J)
5     X(J)=Y(J)+0.5*H*DX(J)
      I=1
      RETURN

      DO 20 J=1,N
      Z(J)=Z(J)+2.0*DX(J)
0     X(J)=Y(J)+0.5*H*DX(J)
      GO TO 25

      DO 30 J=1,N
0     X(J)=Y(J)+(Z(J)+DX(J))*H/6.0

      I=2
      K=0
      RETURN
      END

       THE MILNE'S FORMULA TO PREDICT X(K+1).

      SUBROUTINE PMILNE(M,N,X,DX,T,H)
      DIMENSION X(N),DX(N)

      DO 1 I=1,M
      XT=X(3*M+I)
      X(3*M+I)=X(2*M+I)
      X(2*M+I)=X(M+I)
      X(M+I)=X(I)
      X(I)=XT+4.0*H*(2.0*DX(I)-DX(M+I)+2.0*DX(2*M+I))/3.0
      DX(2*M+I)=DX(M+I)
      DX(M+I)=DX(I)

      T=T+H
      RETURN
```

```
      END

C     THE MILNE'S FORMULA TO CORRECT X(K+1)

      SUBROUTINE CMILNE(M,N,X,DX,T,H)
      DIMENSION X(N),DX(N)

      DO 2 I=1,M
    2 X(I)=X(2*M+I)+H*(DX(I)+4.0*DX(M+I)+DX(2*M+I))/3.0
      RETURN
      END

      THE EVALUATION OF THE COST FUNCTIONAL EQUATION.

      FUNCTION COST(Y,A,B,V)
      DIMENSION Y(88),A(4),B(4),V(2)
      REAL N1,N2,N3,N4
      DATA A1/2.5/,A2/1.0/,A3/0.1/,A4/2.5/,B1/1.0/,B2/1.0/,C1/0.01/,
     *     C2/0.01/,C3/0.01/
      DATA N1,N2,N3,N4,Q1,Q2,Q3,Q4/8*0.001/

      COST=A1*Y(1)**2+A4*Y(4)**2+A2*Y(2)**2+A3*Y(3)**2+C1*Y(5)**2+
     *     C2*Y(6)**2+C3*Y(7)**2+B1*V(1)**2+B2*V(2)**2+N1*A(1)**2+
     *     N2*A(2)**2+N3*A(3)**2+N4*A(4)**2+Q1*B(1)**2+Q2*B(2)**2+
     *     Q3*B(3)**2+Q4*B(4)**2
      RETURN
      END

      INTEGRATION BY SIMPSON'S RULE.

      SUBROUTINE SIMP(F,AREA)
      DIMENSION F(31)
      REAL H,XMAX,XMIN
      DATA H/0.05/,XMAX/0.5/,XMIN/0.0/,N/11/

      H=(XMAX-XMIN)/(N-1)
      X=XMIN+H
      SUM=0.0

      DO 4 I=1,N
      IF(MOD(I,2)) 2,2,3
      SUM=SUM+4.0*F(I)
      GO TO 4
      SUM=SUM+2.0*F(I)
      X=X+H

      AREA=(H/3.0)*(F(1)+SUM+F(N))
      RETURN
      END

      TO SOLVE V1,V2,K1 AND K2.

      SUBROUTINE VVKK(Y,A,B,V,K1,K2)
      DIMENSION Y(88),A(4),B(4),V(2)
      REAL K1,K2
      DATA G2/1.33/,G3/1.42/,CR/1.0/,TG/0.2/,TE/0.2/,B1/1.0/,B2/1.0/

      V(1)=A(1)*Y(1)+A(2)*Y(2)+A(3)*Y(3)+A(4)*Y(4)
      V(2)=B(1)*Y(1)+B(2)*Y(2)+B(3)*Y(3)+B(4)*Y(4)
```

```
 K1=(Y(22)*G2*G3)/TG-2.0*B1*V(1)
 K2=(Y(21)*CR)/TE-2.0*B2*V(2)

 RETURN
 END



  GRAPH OF DELTA VS TIME AT K=1.0
  PROGRAM PLOT1
 DIMENSION X(11),Y(11),ALPH(20),DATA1(11,2)
 REAL ALPH/'FIG.','    ','ROTO','R-AN','GLE ','VS T','IME ','( AT',
*' K= ','1.0 ',' )  ','    ',
*'TIME',' (SE','COND',')   ','ROTO','R-AN','GLE(','DEG)'/
 DATA DATA1/37.1,38.4,42.1,48.0,55.2,62.7,70.0,75.9,79.7,81.5,
*81.1,37.1,38.4,42.1,47.9,54.8,61.9,68.0,72.0,73.5,72.4,68.6/

 ND=11
 NF=1
 KC=3
 HA=0.0
 HB=0.1
 HC=5.0
 VA=0.0
 VB=10
 VC=8.5

 X(1)=0.0
 DO 10 I=2,11
 X(I)=X(I-1)+0.05

 DO 30 I=1,2
 DO 20 J=1,11
 Y(J)=DATA1(J,I)
 CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
 NF=-2
 CONTINUE
 STOP
 END



  GRAPH OF DELTA VS TIME AT K=0.4
  PROGRAM PLOT2
 DIMENSION X(11), Y(11),ALPH(20),DATA2(11,2)
 REAL ALPH/'FIG.','    ','ROTO','R-AN','GLE ','VS T','IME ','( AT',
*' K= ','0.4 ',' )  ','    ',
*'TIME',' (SE','COND',')   ','ROTO','R-AN','GLE(','DEG)'/
 DATA DATA2/37.1,37.6,39.1,41.5,44.3,47.5,50.5,53.1,55.0,56.0,
*      56.1,37.1,37.6,39.1,41.5,44.2,47.1,49.5,51.3,52.1,51.7,50.3/

 ND=11
 NF=1
 KC=3
 HA=0.0
 HB=0.1
 HC=5.0
 VA=0.0
 VB=10
 VC=8.0
```

```
   X(1)=0.0
   DO 10 I=2,11
0  X(I)=X(I-1)+0.05

   DO 30 I=1,2
   DO 20 J=1,11
0  Y(J)=DATA2(J,I)
   CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
   NF=-2
0  CONTINUE
   STOP
   END



    GRAPH OF DELTA VS TIME AT UNSTABLE POINT
    PROGRAM PLOT3
   DIMENSION X(11),Y(11),ALPH(20),DATA3(11,2)
   REAL ALPH/'FIG.','    ','ROTO','R-AN','GLE ','VS T','IME ','( AT',
  *' K= ','2.1 ',' )  ','    ',
  *'TIME',' (SE','COND',')   ','ROTO','R-AN','GLE(','DEG)'/
   DATA DATA3/37.1,39.9,47.8,60.0,75.0,90.0,104.3,117.1,128.0,
  *137.5,146.9,37.1,39.9,47.8,59.8,73.9,90.0,98.9,107.2,110.8,
  *109.8,104.3/

   ND=11
   NF=1
   KC=3
   HA=0.0
   HB=0.1
   HC=5.0
   VA=0.0
   VB=20
   VC=8.0

   X(1)=0.0
   DO 10 I=2,11
0  X(I)=X(I-1)+0.05

   DO 30 I=1,2
   DO 20 J=1,11
0  Y(J)=DATA3(J,I)
   CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
   NF=-2
0  CONTINUE
   STOP
   END



    GRAPH OF DELTA VS TIME FOR K=0.5 BUT USE OPTIMAL AI,BI FOUND
    FOR K=0.2
    PROGRAM PLOT4
   DIMENSION X(11),Y(11),ALPH(20),DATA4(11,2)
   REAL ALPH/'FIG.','    ','ROTO','R-AN','GLE ','VS T','IME ','( AT',
  *' K= ','0.5 ',' )  ','    ',
  *'TIME',' (SE','COND',')   ','ROTO','R-AN','GLE(','DEG)'/
   DATA DATA4/37.1,37.7,39.6,42.5,46.0,49.5,52.6,54.9,55.8,55.3,
  *     53.4,37.1,37.7,39.6,42.5,46.0,49.5,52.6,54.7,55.5,54.5,
```

```
     *      52.7/
C
      ND=11
      NF=1
      KC=3
      HA=0.0
      HB=0.1
      HC=5.0
      VA=0.0
      VB=10
      VC=8.0
C
      X(1)=0.0
      DO 10 I=2,11
   10 X(I)=X(I-1)+0.05
C
      DO 30 I=1,2
      DO 20 J=1,11
   20 Y(J)=DATA4(J,I)
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      NF=-2
   30 CONTINUE
      STOP
      END
C
C
C
C
C
C       GRAPH OF DELTA VS TIME FOR K=0.1 T=0.0 TO T=2.0
C       PROGRAM PLOT5
      DIMENSION X(21),Y(21),ALPH(20),DATA5(21,2)
      REAL ALPH/'FIG.','    ','ROTO','R-AN','GLE ','VS T','IME ','( AT',
     *' K= ','0.1 ',' )  ','    ',
     *'TIME',' (SE','COND',')   ','ROTO','R-AN','GLE(','DEG)'/
      DATA DATA5/37.1,37.6,38.9,40.3,41.2,41.1,39.9,37.7,35.6,34.2,
     *34.0,35.0,36.8,38.5,39.6,39.7,38.8,37.4,36.0,35.1,35.2,
     *37.1,37.6,38.9,40.5,41.7,42.1,41.2,39.0,36.3,34.2,33.5,
     *34.3,36.0,38.2,39.8,40.4,39.7,38.2,36.4,35.2,34.7/
C
      ND=21
      NF=1
      KC=3
      HA=0.0
      HB=0.4
      HC=5.0
      VA=0.0
      VB=10
      VC=7.0
C
      X(1)=0.0
      DO 10 I=2,21
   10 X(I)=X(I-1)+0.1
C
      DO 30 J=1,2
      DO 20 I=1,21
   20 Y(I)=DATA5(I,J)
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      NF=-2
   30 CONTINUE
      STOP
      END
```

```
 GRAPH OF PARAMETERS(AI) VS DISTURBANCE
 PROGRAM PLOT6
DIMENSION X(11),Y(11),ALPH(20),DATA6(11,4)
REAL ALPH/'FIG.','    ','OPTI','MAL ','PARA','METE','RS V',
*'S DI','STUR','BANC','E   ','    ','DIST','URBA','NCE ',
*'(K) ','PARA','METE','RS  ','(AI)'/
DATA DATA6/0.0,-0.012,0.016,0.094,0.172,0.196,0.224,0.250,0.273,
*0.290,0.2999,0.0,-0.242,-0.419,-0.420,-0.416,-0.407,-0.399,-0.392,
*-0.387,-0.382,-0.378,0.0,0.00,-0.001,-0.002,-0.003,-0.002,-0.001,
*0.001,0.003,0.007,0.01,0.0,0.009,-0.016,-0.086,-0.16,-0.184,
*-0.213,-0.242,-0.268,-0.288,-0.303/

ND=11
NF=1
KC=3
HA=0.0
HB=0.2
HC=5.0
VA=-0.5
VB=0.1
VC=8.0

X(1)=0.0
DO 10 I=2,11
X(I)=X(I-1)+0.1

DO 30 I=1,4
DO 20 J=1,11
Y(J)=DATA6(J,I)
NF=I
IF(NF.EQ.4) NF=-4
CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
CONTINUE
STOP
END


 GRAPH OF PARAMETER(BI) VS DISTURBANCE
 PROGRAM PLOT7
DIMENSION X(11),Y(11),ALPH(20),DATA7(11,4)
REAL ALPH/'FIG.','    ','OPTI','MAL ','PARA','METE','RS V',
*'S DI','STUR','BANC','E   ','    ','DIST','URBA','NCE ','(K) ',
*'PARA','METE','RS  ','(BI)'/
DATA DATA7/0.0, 0.0032,-0.0045,-0.0258,-0.048,-0.055,-0.0637,
*-0.0724,
*-0.0804,-0.0867,-0.0917,0.0,0.065,0.1138,0.1163,0.1181,0.1174,
*0.1176,0.1180,0.1185,0.119,0.1198,0.0,0.00,0.0,0.0,0.0,0.0,
*-0.0003,-0.0007,-0.0015,-0.0023,-0.0032,0.0,-0.0024,0.0043,
*0.0235,0.0446,0.0516,0.0606,0.0702,0.0796,0.0872,0.0936/

ND=11
NF=1
KC=3
HA=0.0
HB=0.2
HC=5.0
```

```
      VA=-0.12
      VB=0.03
      VC=8.0

      X(1)=0.0
      DO 10 I=2,11
10    X(I)=X(I-1)+0.1

      DO 30 I=1,4
      DO 20 J=1,11
20    Y(J)=DATA7(J,I)
      NF=I
      IF(NF.EQ.4) NF=-4
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
30    CONTINUE
      STOP
      END


C      GRAPH OF OPTIMUM CONTROLS VS TIME AT K=0.4
C      PROGRAM PLOT8
      DIMENSION X(11),Y(11),ALPHA(20),ALPHB(20),DATA8(11,2)
      REAL ALPHA/'FIG.','    ','OPTI','MUM ','CONT','ROL ','TO T',
     *'URBO','-GEN','ERAT','OR M','ODEL','TIME',' (SE','COND',')   ',
     *'SPEE','DER ','GEAR','    '/
      REAL ALPHB/'FIG.','    ','OPTI','MUM ','CONT','ROL ','TO T','URBO'
     *,'-GEN','ERAT','OR M','ODEL','TIME',' (SE','COND',')   ',
     *'EXCI','TER ','FIEL','D   '/
      DATA DATA8/0.424,0.273,0.148,0.065,0.035,0.066,0.156,0.293,0.456,
     *0.621,0.759,0.738,0.780,0.816,0.840,0.848,0.840,0.814,0.775,0.730,
     *0.683,0.644/

      ND=11
      NF=1
      KC=3
      HA=0.0
      HB=0.1
      HC=5.0
      VA=0.0
      VB=0.2
      VC=4.0

      X(1)=0.0
      DO 10 I=2,11
10    X(I)=X(I-1)+0.05

      DO 20 I=1,11
20    Y(I)=DATA8(I,1)

      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPHA)

      NF=-1
      VA=0.6
      VB=0.1
      VC=3.0

      DO 30 I=1,11
30    Y(I)=DATA8(I,2)
```

```
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPHB)
      STOP
      END
C
C
C
C
C       GRAPH OF MECHANICAL TORQUE AND FIELD VOLTAGE DUE TO OPTIMUM
C       CONTROLS AT K=0.4
C       PROGRAM PLOT9
      DIMENSION X(11),Y(11),ALPHA(20),ALPHB(20),DATA9(11,2)
      REAL ALPHA/'FIG.','    ','VARI','ATIO','N OF',' OPT',
     *'IMUM',' MEC','HANI','CAL ','TORQ','UE  ','TIME',' (SE',
     *'COND',')   ','MECH','ANIC','AL T','ORQU'/
      REAL ALPHB/'FIG.','    ','VARI','ATIO','N OF',' OPT',
     *'IMUM',' FIE','LD V','OLTA','GE  ','    ','TIME',' (SE',
     *'COND',')   ','FIEL','D VO','LTAG','E   '/
      DATA DATA9/0.8,0.799,0.792,0.777,0.752,0.720,0.684,0.651,
     *0.626,0.614,0.617,0.738,0.743,0.755,0.771,0.788,0.801,
     *0.807,0.804,0.792,0.773,0.748/
C
      ND=11
      NF=1
      KC=3
      HA=0.0
      HB=0.1
      HC=5.0
      VA=0.6
      VB=0.05
      VC=4.0
C
      X(1)=0.0
      DO 10 I=2,11
10    X(I)=X(I-1)+0.05
C
      DO 20 I=1,11
20    Y(I)=DATA9(I,1)
C
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPHA)
C
      NF=-1
      VA=0.70
      VB=0.05
      VC=3.0
C
      DO 30 I=1,11
30    Y(I)=DATA9(I,2)
C
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPHB)
      STOP
      END
C
C
C
C
C       GRAPH OF OPTIMUM ANGULAR VELOCITY AT K=0.4
C       PROGRAM PLOT10
      DIMENSION X(11),Y(11),ALPH(20),DATA10(11)
      REAL ALPH/'FIG.','    ','VARI','ATIO','N OF',' OPT','IMUM',
     *' ANG','ULAR',' VEL','OCIT','Y   ','TIME',' (SE','COND',
     *')   ','ANGU','LAR ','VELO','CITY'/
      DATA DATA10/0.0,0.366,0.682,0.907,1.006,0.960,0.768,0.456,
```

```
     *0.069,-0.329,-0.675/
C
      ND=11
      NF=-1
      KC=3
      HA=0.0
      HB=0.1
      HC=5.0
      VA=-0.75
      VB=0.25
      VC=8.0
C
      X(1)=0.0
      DO 10 I=2,11
10    X(I)=X(I-1)+0.05
C
      DO 20 I=1,11
20    Y(I)=DATA10(I)
C
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      STOP
      END
C
C
C
C      GRAPH OF OPTIMUM EXCITATION VOLTAGE AT K=0.4
C      PROGRAM PLOT11
      DIMENSION X(11),Y(11),ALPH(20),DATA11(11)
      REAL ALPH/'FIG.','    ','VARI','ATIO','N OF',' OPT','IMUM',
     *' EXC','ITAT','ION ','VOLT','AGE ','TIME',' (SE','COND',
     *')   ','EXCI','TATI','ON  ','EMF '/
      DATA DATA11/2.889,2.908,2.955,3.018,3.085,3.135,3.159,
     *3.147,3.100,3.026,2.928/
C
      ND=11
      NF=-1
      KC=3
      HA=0.0
      HB=0.1
      HC=5.0
      VA=2.8
      VB=0.05
      VC=8.0
C
      X(1)=0.0
      DO 10 I=2,11
10    X(I)=X(I-1)+0.05
      DO 20 I=1,11
20    Y(I)=DATA11(I)
C
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      STOP
      END
C
C
C
C      GRAPH OF DELTA VS TIME FOR K=0.4 T=0.0 T=2.0
C      PROGRAM PLOT12
      DIMENSION X(21),Y(21),ALPH(20),DATA12(21,2)
      REAL ALPH/'FIG.','    ','ROTO','R-AN','GLE ','VS T','IME ',
```

```
     *'( AT',' K= ','0.4 ',' )  ','     ',
     *'TIME',' (SE','COND',')   ','ROTO','R-AN','GLE(','DEG)'/
      DATA DATA12/37.1,39.1,44.2,49.5,52.1,50.3,44.3,36.1,29.2,25.6,
     *26.4,30.9,37.3,43.2,46.5,45.7,41.8,36.6,32.0,29.8,30.4,
     *37.1,39.1,44.3,50.5,55.0,56.1,51.6,42.3,32.3,25.2,23.0,
     *25.9,32.8,41.0,47.4,49.5,46.8,40.9,34.4,29.7,28.2/
C
      ND=21
      NF=1
      KC=3
      HA=0.0
      HB=0.4
      HC=5.0
      VA=0.0
      VB=10.0
      VC=7.0
C
      X(1)=0.0
      DO 10 I=2,21
10    X(I)=X(I-1)+0.1
C
      DO 30 I=1,2
      DO 20 J=1,21
20    Y(J)=DATA12(J,I)
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      NF=-2
30    CONTINUE
      STOP
      END
C
C
C
C      PHASE PLANE PLOT AT K=2.1
C      PROGRAM PLOT13
      DIMENSION X(21),Y(21),ALPH(20),DATA13(21,4)
      REAL ALPH/'FIG.','    ','PHAS','E PL','ANE ','PLOT',
     *' AT ',' K= ','0.4 ','    ','    ','    ',
     *'ROTO','R-AN','GLE(','DEG)','ANGU','LAR-','VELO','CITY'/
      DATA DATA13/37.1,39.1,44.2,49.5,52.1,50.3,44.3,36.1,29.2,
     *25.6,26.4,30.9,37.3,43.2,46.5,45.7,41.8,36.6,32.0,29.8,30.4,
     *37.1,39.1,44.3,50.5,55.0,56.1,51.6,42.3,32.3,25.2,23.0,
     *25.9,32.8,41.0,47.4,49.5,46.8,40.9,34.4,29.7,28.2,
     *0.0,0.68,1.01,0.77,0.07,-0.67,-1.34,-1.40,-0.96,-0.24,0.49,
     *1.02,1.15,0.85,0.22,-0.45,-0.87,-0.91,-0.62,-0.14,0.34,
     *0.0,0.69,1.07,0.99,0.51,-0.14,-1.32,-1.82,-1.57,-0.83,0.09,
     *0.91,1.39,1.36,0.79,-0.07,-0.82,-1.17,-1.03,-0.56,0.06/
C
      ND=21
      NF=1
      KC=3
      HA=10.0
      HB=10.0
      HC=5.0
      VA=-2.0
      VB=0.5
      VC=8.0
C
      DO 30 J=1,2
      DO 10 I=1,21
10    X(I)=DATA13(I,J)
```

```
      DO 20 I=1,21
0     Y(I)=DATA13(I,J+2)
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      NF=-2
0     CONTINUE
      STOP
      END



C     PHASE PLANE PLOT AT K=2.1
      PROGRAM PLOT14
      DIMENSION X(11),Y(11),ALPH(20),DATA14(11,4)
      REAL ALPH/'FIG.','    ','PHAS','E PL','ANE ','PLOT',' AT ',
     *' K= ','2.1 ','    ','    ','    ',
     *'ROTO','R-AN','GLE(','DEG)','ANGU','LAR-','VELO','CITY'/
      DATA DATA14/37.1,39.9,47.8,59.8,73.9,90.0,98.9,
     *107.2,110.8,109.8,104.3,
     *37.1,39.9,47.8,60.0,75.0,90.0,104.3,117.1,128.0,137.5,146.9,
     *0.0,1.92,3.57,4.68,5.04,4.59,3.49,2.03,0.43,-1.14,-2.55,
     *0.0,1.93,3.60,4.81,5.40,5.32,4.77,4.07,3.49,3.22,3.39/

      ND=11
      NF=1
      KC=3
      HA=0.0
      HB=30.0
      HC=5.0
      VA=-3.0
      VB=1.0
      VC=8.5

      DO 30 J=1,2
      DO 10 I=1,11
0     X(I)=DATA14(I,J)
      DO 20 I=1,11
0     Y(I)=DATA14(I,J+2)
      CALL CGPL2(X,Y,ND,NF,KC,HA,HB,HC,VA,VB,VC,ALPH)
      NF=-2
0     CONTINUE
      STOP
      END
```